Properties of Exponential Functions

THEOREM Laws of Exponents

If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

- $a^s \cdot a^t = a^{s+t}$
- $(a^s)^t = a^{st}$
- $(ab)^s = a^s \cdot b^s$
- $1^s = 1$
- $a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s$
- $a^0 = 1$

(1)

DEFINITION Exponential Function

An **exponential function** is a function of the form

$$f(x) = Ca^x$$

where a is a positive real number $(a > 0)$, $a \neq 1$, and $C \neq 0$ is a real number. The domain of f is the set of all real numbers. The base a is the **growth factor**, and, because $f(0) = Ca^0 = C$, C is called the **initial value**.

THEOREM

For an exponential function $f(x) = Ca^x$, $a > 0$, $a \neq 1$, and $C \neq 0$, if x is any real number, then

$$\frac{f(x+1)}{f(x)} = a \quad \text{or} \quad f(x+1) = af(x)$$

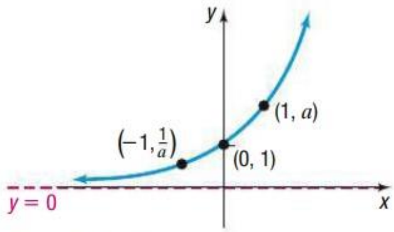

Figure 21 $f(x) = a^x, a > 1$

Properties of the Exponential Function $f(x) = a^x, a > 1$

- The domain is the set of all real numbers, or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers, or $(0, \infty)$ using interval notation.
- There are no x-intercepts; the y-intercept is 1.
- The x-axis $(y = 0)$ is a horizontal asymptote of the graph of f as $x \to -\infty$.
- $f(x) = a^x, a > 1$, is an increasing function and is one-to-one.
- The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$ and $(1, a)$.
- The graph of f is smooth and continuous, with no corners or gaps. See Figure 21.

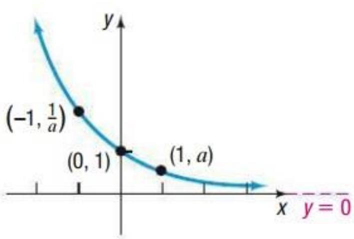

Figure 25 $f(x) = a^x, 0 < a < 1$

Properties of the Exponential Function $f(x) = a^x, 0 < a < 1$

- The domain is the set of all real numbers, or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers, or $(0, \infty)$ using interval notation.
- There are no x-intercepts; the y-intercept is 1.
- The x-axis $(y = 0)$ is a horizontal asymptote of the graph of f as $x \to \infty$.
- $f(x) = a^x, 0 < a < 1$, is a decreasing function and is one-to-one.
- The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
- The graph of f is smooth and continuous, with no corners or gaps. See Figure 25.

DEFINITION Number e

The **number *e*** is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n \qquad \textbf{(2)}$$

approaches as $n \to \infty$. In calculus, this is expressed, using limit notation, as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\text{If} \quad a^u = a^v, \quad \text{then} \quad u = v. \qquad \textbf{(3)}$$

SUMMARY

Properties of the Exponential Function

$f(x) = a^x, \quad a > 1$

- Domain: the interval $(-\infty, \infty)$; range: the interval $(0, \infty)$
- x-intercepts: none; y-intercept: 1
- Horizontal asymptote: x-axis $(y = 0)$ as $x \to -\infty$
- Increasing; one-to-one; smooth; continuous
- See Figure 21 for a typical graph.

$f(x) = a^x, \quad 0 < a < 1$

- Domain: the interval $(-\infty, \infty)$; range: the interval $(0, \infty)$
- x-intercepts: none; y-intercept: 1
- Horizontal asymptote: x-axis $(y = 0)$ as $x \to \infty$
- Decreasing; one-to-one; smooth; continuous
- See Figure 25 for a typical graph.

If $a^u = a^v$, then $u = v$.