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Introduction to Logic

Chapter 3

MAT 120

3.1 Statements & Quantifiers

Negations of Quantified Statements

Statement		Negation
All do	$\longleftrightarrow$	Some do not
Some do	$\longleftrightarrow$	None do

3.2 Truth Tables & Equivalent Statements

Truth Table for Conjunction

p and q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for Disjunction

p or q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Common Translations of $p \to q$

If p, then q	p is sufficent for q
If p, q	q is necessary for p
p implies q	All p are q
p only if q	q if p

Biconditionals

$$p \leftrightarrow q \equiv (q \to p) \wedge (p \to q)$$

Truth Table for Biconditional

p if and only if q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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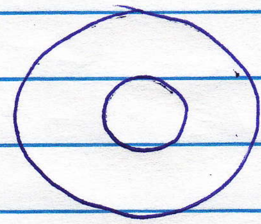
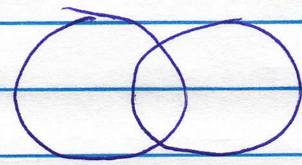
3.5 Analyzing Arguments with Euler Diagrams

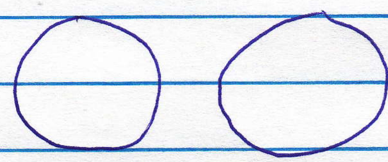
Case 1 "All"........



Case 2 "Some"........



Case 3 "No" or "None"......



3.6 Analyzing Arguments with Truth Tables

Step 1 Assign letter to represent each component statement.

Step 2 Express each premise & conclusion symbolically.

Ex:

Premise 1: $p \to q$
Premise 2: p
Conclusion: q

Step 3 Form symbolic statement of entire argument. Write conjunction ($\wedge$) of all premises as antecedent of a conditional statement and conclusion of argument as consequent.

Ex:

$$[\underset{\text{premise}}{(p \to q)} \underset{\text{and}}{\wedge} \underset{\text{premise}}{p}] \underset{\text{implies}}{\to} \underset{\text{conclusion}}{q}$$

Step 4 Complete truth table. If it is a tautology, argument is valid. Otherwise invalid.

Ex:

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$

Valid Argument Forms

Modus Ponens	Modus Tollens	Disjunctive Syllogism	Reasoning by Transitivity
$p \rightarrow q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow q$
p	$\sim q$	$\sim p$	$q \rightarrow r$
q	$\sim p$	q	$p \rightarrow r$

Invalid Argument Forms (Fallacies)

Fallacy of the Converse	Fallacy of the Inverse
$p \rightarrow q$	$p \rightarrow q$
q	$\sim p$
p	$\sim q$