Signed Numbers Workbook

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Table of Contents

Introduction

Hello and welcome to my *Signed Numbers Workbook*. I am confident that after working through this resource, your proficiency will improve when solving math problems with signed numbers.

Understanding and correctly applying the rules of signed numbers is crucial for successfully completing the introduction to algebra section, which is the entire second half of our MAT 050 course. Moreover, the manipulation of signed numbers will likely be revisited in a future algebra-based math course that may be required for your major.

The primary aim of this workbook is to increase your *accuracy* first, and *speed* second, when carrying out signed number **operations** $(+, -, \times, \div)$. This can only be accomplished if you read through the material, pause to reflect as needed, then complete the drill problems listed on page 8. You must demonstrate mastery of what you learned to prove to yourself that you fully grasp the concepts presented.

Most of the problems will not take long to finish given that they involve operations of two signed numbers. First write all of your answers on the workbook. After all problems are done, compare them to the answers listed at the bottom of page 8. Do not look at the answers until you first complete all 20 problems. Consider the problems as a Quiz whereby a minimum of a 90% score (18 of 20 correct) signifies competence.

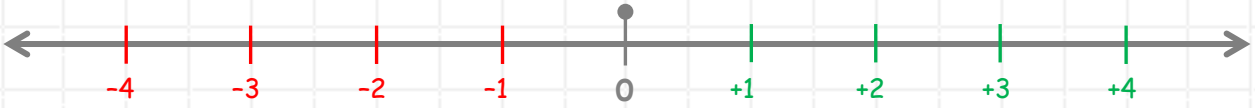
The time and effort that you invest in internalizing the techniques explained in this workbook is well worth it because you will have acquired an essential skill in your introduction to algebra.

As a final point, I will not wish you "good luck" with your mathematics studies. The reason being is that having good luck has little to do with math aptitude! Therefore, I encourage you instead to cultivate…

"Good Skills"-**George Hartas**

Anatomy of a Signed Number

Before examining the rules of signed numbers, it is helpful to first ask, "What are signed numbers?" They are numbers you have already used but with a positive "+" or negative "–" sign in front of them. Signed numbers sit a certain distance, to the left and to the right, from zero on a number line.



Although the number line concept provides a visual representation of signed numbers, we will not be using it in this workbook. The four rules of signed numbers will be utilized instead. So let's begin.

The sign that is directly in front of a number (to its immediate left) is part of that number. Think of the sign and number as being coupled, or glued, together. The two cannot be separated. Thus, a **signed number** contains two parts: (1) sign, and (2) number.

If there are two signed numbers in a row, think of them as simply floating next to each other. For example, if a problem had $-3 + 2$, we would have two signed numbers. One signed number is -3 and the other is $+2$.

If we ever wanted to move a signed number around to another location (Commutative Property), the sign would also have to move along with the number. This is because the sign and the number belong together – they are a couple. For example, if we had the problem $-3 + 2 - 4$, we could move the -3 from the front to the back if we wanted.

$-3 + 2 - 4$ $\quad\Rightarrow\quad$ $2 - 4 - 3$

Notice that the "−" got moved along with the number 3. Also, the "+" in front of the 2 is not needed now.

CAUTION: A sign to the *right* of a number is **not** part of that number. Instead, that sign belongs to the number to the right of that sign. From left to right, the order of a sign-number couple is: sign first, number second.

OBJECTIVE 1 - Multiplication and Division

Rules 1 and 2 apply for both multiplication and division.

RULE 1: If signs are the same, result is positive.

Ex 1: $(+2)(+1) = (+2)$ simplifies to $2 \cdot 1 = 2$

Ex 2: $(-2)(-1) = (+2)$ simplifies to $-2(-1) = 2$

A number with no sign makes it positive. The "•" symbol can replace parenthesis.

The parenthesis for (-2) can be removed since there is another parenthesis to the right of -2. **CAUTION:** That other parenthesis (-1) however **must** stay to indicate multiplication between the two signed numbers.

Division also follows Rule 1:

Ex 3: $(-2) \div (-1) = (+2)$ or as $\frac{(-2)}{(-1)} = (+2)$ simplifies to $\frac{-2}{-1} = 2$

CAUTION: Multiplication can be shown in three ways: (1) parenthesis "( )", (2) the "•" symbol, or (3) less often, the "×" symbol. If you do not see these around (or next to) a signed number, Rules 1 and 2 **do not** apply.

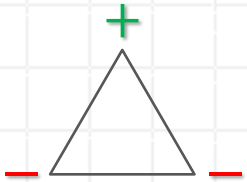
RULE 2: If signs are different, result is negative.

Ex 4: $(-2)(+1) = (-2)$ simplifies to $-2 \cdot 1 = -2$

Ex 5: $(+2)(-1) = (-2)$ simplifies to $2(-1) = -2$

It does not matter which number has the negative sign, the result is still negative.

Division also follows Rule 2.



Aids for Both × and ÷

+	–	–
–	+	–
–	–	+

Signs Triangle
1. Start at any sign.
2. Go clockwise or counterclockwise.
3. Last sign gives result.

Signs Grid
1. Start at any sign except middle "+".
2. Go across, down, or diagonal.
3. Last sign gives result.

CAUTION: Avoid using phrases like, "A negative AND a negative is…" because it is not clear what the word **AND** implies. Are we multiplying two negative numbers, or are we adding two negative numbers? Instead, say "A negative TIMES a negative is…" for multiplication.

OBJECTIVE 2 - Addition and Subtraction

It can be unclear as to whether we should add or subtract signed numbers. Instead, let's just "combine" them using Rules 3 or 4. We will then add or subtract by the appropriate rule, which depends on the sign of the numbers.

RULE 3: **If signs are the same: (1) add the numbers, (2) result uses same sign.**

Ex 6: $+2+1=+3$ simplifies to $2+1=3$

Ex 7: $-2-1=-3$

For both Example 6 and 7, the sign of the two numbers are the same. Add the numbers and use that same sign for the result.

If the signs are the same, we must "gather up" or "collect" all of the positives (if both signs are positive) or all of the negatives (if both signs are negative). We need to **add** so that we can count up those numbers to find their total. Thus, the result uses the **same sign** as the numbers that we summed.

RULE 4: **If signs are different: (1) subtract the numbers, (2) result uses sign of bigger number.**

Ex 8: $+2-1=+1$ simplifies to $2-1=1$

Ex 9: $-2+1=-1$

For both Example 8 and 9, the sign of the two numbers are different. Subtract the numbers and use sign of bigger number for the result.

If the signs are **different**, we must find out *how much* different the two numbers are from each other. We can find out how different they are by "taking the difference", which means to **subtract** those numbers.

If the positive number is bigger than the negative number, the result will be positive since there are more positives compared to negatives.

If the negative number is bigger than the positive number, the result will be negative since there are more negatives compared to positives.

WAR OF THE SIGNS - *an alternate perspective.* A battle rages between positives and negatives. They try to eliminate each other. The sign allied with the biggest number wins. Winning sign coupled with the difference between the two numbers are "paraded" in result.

OBJECTIVE 3 - Short Problems Combining Multiplication with Addition/Subtraction

Another Format When Adding or Subtracting Signed Numbers

Sometimes multiplication is *implied* when solving "addition" or "subtraction" problems as in, $-2-(-1)$. We know we must multiply since: (1) there are two consecutive signs (two "$-$"), and furthermore, (2) the problem has a parenthesis. Recall from Rule 1 and 2 that parenthesis means multiplication.

Whenever a problem has two consecutive signs, first "multiply out" the two signs using Rule 1 or 2. After multiplying, there will only be one sign remaining instead of the two original signs. The parenthesis too will be removed.

Therefore, multiplying *first* makes the problem easier to solve because we can then "combine" the two numbers (adding or subtracting) using Rule 3 or 4.

Ex 10: $-2-(-1)$ simplifies to $-2+1$ with result as -1

Reduce two signs down to one by "multiplying" the consecutive signs: $-2-(-1)$. Problem is now simpler to solve: $-2+1$.

Sign of -2 and $+1$ are different. **Rule 4** says to (1) subtract numbers, (2) use sign of bigger number. The result is -1.

Ex 11: $-2+(-1)$ simplifies to $-2-1$ with result as -3

Simplify problem by "multiplying" the two consecutive signs: $-2+(-1)$. The problem becomes: $-2-1$.

Sign of -2 and -1 are the same. **Rule 3** says to (1) add numbers, (2) use same sign. Result is: -3.

Ex 12: $2+(-1)$ simplifies to $2-1$ with result as 1

No sign in front of a number means it is "$+$". Thus, the leading number is $+2$. Multiply the two consecutive signs $2+(-1)$ to reduce the signs down to one. The problem is now much easier to solve: $2-1$.

Sign of $+2$ and -1 are different. **Rule 4** says to (1) subtract numbers, (2) use sign of bigger number. The result is $+1$, or just 1.

OBJECTIVE 4 - Long Problems Combining Multiplication with Addition/Subtraction

There are some problems you have to solve that contain several signed numbers. For example, using the four rules of signs, you must be able to solve an expression like: $-5-(-2)-3-6(7)-9-(-4)$.

First we need to review an algebra concept called *The Order of Operations.* As a memory aid, we will use the acronym **PEMDAS**.

Highest priority.

P = **P**arenthesis. Simplify expression *inside* a parenthesis.

E = **E**xponents. Simplify exponents (powers).

M = **M**ultiplication. Multiply signed numbers.

D = **D**ivision. Divide signed numbers.

A = **A**ddition. Add signed numbers.

S = **S**ubtraction. Subtract signed numbers.

Lowest priority.

This mnemonic might help you remember the *order* of the letters in PEMDAS: "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".

Order matters for PEMDAS. If the problem has a parenthesis, the "P" must be done *first.* If the problem has an exponent, the "E" must be done *second,* and so on. PEMDAS helps us to solve complicated expressions because it guides us into doing the correct operation for that step in the problem.

Let's solve the problem above with PEMDAS and the four rules of signs. So where do we start? Let's use PEMDAS as a guide. For the "P", can the inside of the parenthesis be further simplified? No. Next is "E". Do we have any visible exponents to simplify? No. Next is "M". Do we have any multiplication anywhere? Yes, and in fact three places. Therefore, multiply *first!*

Ex 13: $-5-(-2)-3-6(7)-9-(-4)$

$-5\quad+2\quad-3\quad-42\quad-9\quad+4$

$-3-3-42-9+4$

$-6-42-9+4$

$-48-9+4$

$-57+4$

-53

Notice the -5, -3, and -9 are **not** multiplying with any number since they are not up against a parenthesis to their *right.* They are floating next to other numbers; just hanging out…

Multiplying simplifies these:
$-(-2)$ simplifies to $+2$
$-6(7)$ simplifies to -42
$-(-4)$ simplifies to $+4$

Complete the problem in multiple steps using an inverted pyramid format. Keep "combining" the two leftmost numbers with **Rules 3 and 4** until you arrive at the result.

Problems and Answers

YOUR TURN: It is time to show your mastery of signed number operations. Write all results then check your answers. **Goal:** 90% (18 of 20 correct).

OBJECTIVE 1 Problems - Multiplication and Division

P. 1: $-4(-3) =$ _____ P. 2: $12 \div (-2) =$ _____ P. 3: $-7 \cdot 6 =$ _____

P. 4: $-16 \div 4 =$ _____ P. 5: $6(-5) =$ _____ P. 6: $-35 \div (-7) =$ _____

OBJECTIVE 2 Problems - Addition and Subtraction

P. 7: $-7 - 2 =$ _____ P. 8: $13 - 6 =$ _____ P. 9: $-9 + 11 =$ _____

P. 10: $-8 + 5 =$ _____ P. 11: $-16 - 16 =$ _____ P. 12: $-17 + 17 =$ _____

OBJECTIVE 3 Problems - Short Problems Combining Multiplication with Addition/Subtraction

P. 13: $-7 + (-4) =$ _____ P. 14: $18 - (-11) =$ _____ P. 15: $-1 - (+1) =$ _____

P. 16: $14 + (-8) =$ _____ P. 17: $-5 - (-5) =$ _____ P. 18: $0 + (-10) =$ _____

OBJECTIVE 4 Problems - Long Problems Combining Multiplication with Addition/Subtraction

P. 19: $7 + (-1) - 8 - 9 - 2(-3) =$ _____ P. 20: $-9(-3) + 5(-6) - 11 - (-7) =$ _____

Do not look at the answers below until you complete all of the problems above.

Answers

P. 1: 12	P. 2: -6	P. 3: -42	P. 4: -4	P. 5: -30
P. 6: 5	P. 7: -9	P. 8: 7	P. 9: 2	P. 10: -3
P. 11: -32	P. 12: 0	P. 13: -11	P. 14: 29	P. 15: -2
P. 16: 6	P. 17: 0	P. 18: -10	P. 19: -5	P. 20: -7