

MULTIPLICATION FACTS WORKBOOK

Introduction

- Developing an **accurate** and **quick** recall of the multiplication facts is probably the most important skill you will develop in *MAT 025 - Arithmetic Review*.
- You must have already mastered the multiplication facts by the time you start MAT 050.
 - If you had not...
- Multiplication is represented symbolically in three ways:
 - Traditional ' \times '.
 - Multiplication dot ' \cdot '.
 - Parentheses ' $()$ '.
- Numbers that are multiplied together are called **factors**.
- The answer to a multiplication problem is called a **product**.

Even and Odd Numbers

- An even number has a right-hand digit of **0, 2, 4, 6, 8**.
- An odd number has a right-hand digit of **1, 3, 5, 7, 9**.
- When an *even* number is multiplied by:
 - An *even* number, the product is **even**. Ex: $4 \times 6 = 24$.
 - An *odd* number, the product is **even**. Ex: $4 \times 5 = 20$.
- When an *odd* number is multiplied by:
 - An *even* number, the product is **even** (same as above). Ex: $5 \times 4 = 20$.
 - An *odd* number, the product is **odd**. Ex: $7 \times 9 = 63$.

Multiplication Facts Drills

- Work on accuracy first, and speed second.
 - The ideal **accuracy** is **100%** correct recall.
 - The ideal **speed** is a **2 second** recall, maximum.
- Start your study with the **List Format** multiplication table ([pages 3-4](#)).
- Next move to the **Skip Counting** multiplication table ([pages 5-6](#)).
- Then master the **Grid Format** multiplication table ([pages 7-8](#)).
- Use the [Math Worksheets](#) from my web page (georgehartas.com/mat-025/#Mod0M0.6_Worksheets).
- Use the [Facts Math Trainer](#) (mathsisfun.com/numbers/math-trainer-multiply.html).
- Use *free* multiplication [facts apps](#) and [flash cards](#). *Be careful* of what you install on your phone.
- Study the multiplication facts daily:
 - Do in the morning, sometime during the day, and at night.
 - Do for 10-15 minutes each session.
 - Continue your drills until you mastered the multiplication facts.

Properties of Multiplication

Repeated Addition

$$3 + 3 + 3 + 3 = 12$$

Adding the same number repeatedly.

- The Multiplication Facts Table simply shows the **answer** to some **question**.
- The question above is: “How many total boxes do we have from four groups of three boxes?”
- And the answer shown in the Multiplication Facts Table is **12**, because the four groups times the three boxes gives us 12.
- This is multiplication because we repeatedly add each group of three boxes four **times** over.
 - Thus, the Multiplication Facts Table is also known as the “**times table**.”
- Multiplication is a time saving summary for repeated addition.

Commutative Property

$$3 \times 4$$

$$3 \text{ groups of } 4 = 12$$

=====

$$4 \times 3$$

$$4 \text{ groups of } 3 = 12$$

- The **Commutative Property of Multiplication** states that we can change the order of the numbers being multiplied and the answer will be the same. From above:
 - $3 \times 4 = 12$ or $4 \times 3 = 12$.
- This property is important for internalizing (memorizing) the Multiplication Facts Table.
- The property implies that it is not necessary to memorize the entire Multiplication Facts Table, but only half of it.

Multiplication Facts Table

(List Format, Filled-In)

1 Times Table	2 Times Table	3 Times Table	4 Times Table
1 X 1 = 1	2 X 1 = 2	3 X 1 = 3	4 X 1 = 4
1 X 2 = 2	2 X 2 = 4	3 X 2 = 6	4 X 2 = 8
1 X 3 = 3	2 X 3 = 6	3 X 3 = 9	4 X 3 = 12
1 X 4 = 4	2 X 4 = 8	3 X 4 = 12	4 X 4 = 16
1 X 5 = 5	2 X 5 = 10	3 X 5 = 15	4 X 5 = 20
1 X 6 = 6	2 X 6 = 12	3 X 6 = 18	4 X 6 = 24
1 X 7 = 7	2 X 7 = 14	3 X 7 = 21	4 X 7 = 28
1 X 8 = 8	2 X 8 = 16	3 X 8 = 24	4 X 8 = 32
1 X 9 = 9	2 X 9 = 18	3 X 9 = 27	4 X 9 = 36
1 X 10 = 10	2 X 10 = 20	3 X 10 = 30	4 X 10 = 40
1 X 11 = 11	2 X 11 = 22	3 X 11 = 33	4 X 11 = 44
1 X 12 = 12	2 X 12 = 24	3 X 12 = 36	4 X 12 = 48
5 Times Table	6 Times Table	7 Times Table	8 Times Table
5 X 1 = 5	6 X 1 = 6	7 X 1 = 7	8 X 1 = 8
5 X 2 = 10	6 X 2 = 12	7 X 2 = 14	8 X 2 = 16
5 X 3 = 15	6 X 3 = 18	7 X 3 = 21	8 X 3 = 24
5 X 4 = 20	6 X 4 = 24	7 X 4 = 28	8 X 4 = 32
5 X 5 = 25	6 X 5 = 30	7 X 5 = 35	8 X 5 = 40
5 X 6 = 30	6 X 6 = 36	7 X 6 = 42	8 X 6 = 48
5 X 7 = 35	6 X 7 = 42	7 X 7 = 49	8 X 7 = 56
5 X 8 = 40	6 X 8 = 48	7 X 8 = 56	8 X 8 = 64
5 X 9 = 45	6 X 9 = 54	7 X 9 = 63	8 X 9 = 72
5 X 10 = 50	6 X 10 = 60	7 X 10 = 70	8 X 10 = 80
5 X 11 = 55	6 X 11 = 66	7 X 11 = 77	8 X 11 = 88
5 X 12 = 60	6 X 12 = 72	7 X 12 = 84	8 X 12 = 96
9 Times Table	10 Times Table	11 Times Table	12 Times Table
9 X 1 = 9	10 X 1 = 10	11 X 1 = 11	12 X 1 = 12
9 X 2 = 18	10 X 2 = 20	11 X 2 = 22	12 X 2 = 24
9 X 3 = 27	10 X 3 = 30	11 X 3 = 33	12 X 3 = 36
9 X 4 = 36	10 X 4 = 40	11 X 4 = 44	12 X 4 = 48
9 X 5 = 45	10 X 5 = 50	11 X 5 = 55	12 X 5 = 60
9 X 6 = 54	10 X 6 = 60	11 X 6 = 66	12 X 6 = 72
9 X 7 = 63	10 X 7 = 70	11 X 7 = 77	12 X 7 = 84
9 X 8 = 72	10 X 8 = 80	11 X 8 = 88	12 X 8 = 96
9 X 9 = 81	10 X 9 = 90	11 X 9 = 99	12 X 9 = 108
9 X 10 = 90	10 X 10 = 100	11 X 10 = 110	12 X 10 = 120
9 X 11 = 99	10 X 11 = 110	11 X 11 = 121	12 X 11 = 132
9 X 12 = 108	10 X 12 = 120	11 X 12 = 132	12 X 12 = 144

- The *List Format* is a good place to begin if you are struggling with the multiplication facts.
- Notice the built-in repeated addition pattern for each number being multiplied.

Multiplication Facts Table

(List Format, Blank)

1 x 2 = 1 x 3 = 1 x 4 = 1 x 5 = 1 x 6 = 1 x 7 = 1 x 8 = 1 x 9 = 1 x 10 = 1 x 11 = 1 x 12 =	2 x 2 = 2 x 3 = 2 x 4 = 2 x 5 = 2 x 6 = 2 x 7 = 2 x 8 = 2 x 9 = 2 x 10 = 2 x 11 = 2 x 12 =	3 x 2 = 3 x 3 = 3 x 4 = 3 x 5 = 3 x 6 = 3 x 7 = 3 x 8 = 3 x 9 = 3 x 10 = 3 x 11 = 3 x 12 =	4 x 2 = 4 x 3 = 4 x 4 = 4 x 5 = 4 x 6 = 4 x 7 = 4 x 8 = 4 x 9 = 4 x 10 = 4 x 11 = 4 x 12 =
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9 x 2 = 9 x 3 = 9 x 4 = 9 x 5 = 9 x 6 = 9 x 7 = 9 x 8 = 9 x 9 = 9 x 10 = 9 x 11 = 9 x 12 =	10 x 2 = 10 x 3 = 10 x 4 = 10 x 5 = 10 x 6 = 10 x 7 = 10 x 8 = 10 x 9 = 10 x 10 = 10 x 11 = 10 x 12 =	11 x 2 = 11 x 3 = 11 x 4 = 11 x 5 = 11 x 6 = 11 x 7 = 11 x 8 = 11 x 9 = 11 x 10 = 11 x 11 = 11 x 12 =	12 x 2 = 12 x 3 = 12 x 4 = 12 x 5 = 12 x 6 = 12 x 7 = 12 x 8 = 12 x 9 = 12 x 10 = 12 x 11 = 12 x 12 =

- Start with the smaller easy numbers and work your way up to 12.

Multiplication Facts Table
(Skip Counting Format, Filled-In)

Skip Counting 2-12, Filled-In

Count by: ↓	Multiples of: ⇨	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

- Start by choosing any number listed under the **Count by** column.
- Then go from left to right and under the **Multiples of** column, notice that the number gets bigger by the same amount for each multiple.
- The *Skip Counting Format* is like the *List Format*, except that this list is horizontal.
 - The repeated addition pattern is even more evident in this table.
- The *Skip Counting Format* looks like the traditional *Multiplication Facts Table (Grid Format)*.
 - In fact, it is the same with some minor variations.

Multiplication Facts Table
(Skip Counting Format, Blank)

Skip Counting 2-12, Blank

<i>Count by:</i> ↓	<i>Multiples of:</i> ⇨	2	3	4	5	6	7	8	9	10	11	12
2	2	4										
3	3	6										
4	4	8										
5	5	10										
6	6	12										
7	7	14										
8	8	16										
9	9	18										
10	10	20										
11	11	22										
12	12	24										

- The first two multiples are given so that you can see the pattern of repeated addition.
- Fill out the table by repeatedly adding a number listed under the *Count by* column from left to right.
- Pay attention to the *Multiples of* number at the top as you fill out each multiple of a number.
 - This is an important step for creating a link between:
 - The number you chose (under the *Count by* column).
 - The multiple of that number (the *Multiples of* number at the top)
 - The answer you write on the table (the product of *Count by* and *Multiples of*).
 - Developing this link using the *Skip Counting Format* will make it easier for you to master the traditional *Grid Format*.
- Start by learning the multiples of a number **in order**.
- Eventually internalize the multiples of that number **out of order**.
- Then you will be ready to graduate to the *Grid Format* shown next.

Multiplication Facts Table

(Grid Format, Filled-In)

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

- The traditional *Multiplication Facts Table (Grid Format)* is the “gold standard” to internalize.
- Choose two numbers (factors) to multiply: one number from the left shaded column and one number from the top shaded row.
- The cell location on the grid where these two numbers intersect (meet) is the answer (product) of multiplying these two numbers.
- The grid is split in half by the downward diagonal shaded region.
 - Thanks to the *Commutative Property of Multiplication*, the top-right half of the grid is a mirror image of the bottom-left half of the grid.
 - Therefore, you only need to learn half of the grid. Recall earlier that:
 - $3 \times 4 = 12$.
 - $4 \times 3 = 12$.
 - The diagonal shaded region represents the product when multiplying by the *same* number, called the **square** of a number (or *perfect square*). Knowing these will help your recall.
 - $4 \times 4 = 16$.
 - $8 \times 8 = 64$.
- Spend whatever *time* you need in studying this grid to internalize the multiplication facts.

Multiplication Facts Table

(Grid Format, Blank)

X	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

- You can fill out the grid from left to right (by rows) or from top to bottom (by columns).
 - I think it is more intuitive to go from left to right across a row.
- As you fill out the grid from left to right, avoid using skip counting, exclusively.
 - Instead, start to program your mind that “2 times 6 is 12.”
 - For example, let’s say we are multiplying by 2’s going from left to right.
 - We start with “2 times 1 is 2” then “2 times 2 is 4” then “2 times 3 is 6” then “2 times 4 is 8” then “2 times 5 is 10” then “2 times 6 is 12”, etc.
 - Rather than just skip counting on the grid, mentally link the two numbers being multiplied.
- Zeros are not listed because 0 times any number is 0. Easy, right?
- Although 1 is listed on the grid, it too is easy. One times any number is that other number.
- Internalize the multiplication facts in the *following order*, listed from **easy to difficult**:
 - Start learning 2, 10, 11, 5.
 - Then learn 3, 4.
 - Then learn 9.
 - Then learn 6, 7, 8.
 - Finally, learn 12.

Properties and Techniques of the Numbers

(Listed from easy to difficult)

Number 2

Properties

- The number 2 is even.

Techniques

- To multiply by 2, add the number to itself. Double it.
- 2×9 is same as $9 + 9 = 18$.

Number 10

Properties

- The number 10 is even.

Techniques

- When multiplying by 10, the right-hand digit of the product is **0**. Ex: $10 \times 7 = 70$.

Number 11

Properties

- The number 11 is odd.

Techniques

- When multiplying by 11 with a number 1-9, that other number repeats in the product.
 - $11 \times 5 = 55$.
- When multiplying 11 by either 10, 11, or 12 (up to 18), add the digits of the other number and place the sum between the bookend digits of the original problem.
 - Example 11×11 :
 - **Bookend digits** are 11×11 .
 - Sum the digits of **other number**, 11×11 , so $1 + 1 = 2$.
 - Place the sum between the bookend digits, $1(1 + 1)1 = 121$.
 - Example 11×12 :
 - **Bookend digits** are 11×12 .
 - Sum the digits of **other number**, 11×12 , so $1 + 2 = 3$.
 - Place the sum between the bookend digits, $1(1 + 2)2 = 132$.

Number 5

Properties

- The number 5 is odd.
- When multiplying by 5, the right-hand digit of the product is either **0** or **5**.

Techniques

- If 5 is multiplied by an **even** number, cut the even number in half then times 10.
 - 5×6 so cut 6 in half to get 3, then times 10 to get **30**.
- If 5 is multiplied by an **odd** number, times the odd number by 10, then cut the answer in half.
 - 5×9 so $9 \times 10 = 90$, then cut answer in half to get **45**.

Number 3

Properties

- The number 3 is odd.
- When multiplying by 3, then adding digits of the product, the sum alternates from **3**, **6**, and **9**.
 - $3 \times 1 = \mathbf{3}$.
 - $3 \times 2 = \mathbf{6}$.
 - $3 \times 3 = \mathbf{9}$.
 - $3 \times 4 = 12$ then $1 + 2 = \mathbf{3}$.
 - $3 \times 5 = 15$ then $1 + 5 = \mathbf{6}$.
 - $3 \times 6 = 18$ then $1 + 8 = \mathbf{9}$.

Techniques

- Multiplying by 3 is like multiplying by **2**, then adding one more of the **other number**.
 - 3×4 is same as $\mathbf{2} \times 4 + \mathbf{4} = \mathbf{12}$.
 - 3×9 is same as $\mathbf{2} \times 9 + \mathbf{9} = \mathbf{27}$.

Number 4

Properties

- The number 4 is even.

Techniques

- To multiply by 4, double the number, and then double again. A double-double.
 - 4×9 is the same as $\mathbf{9} + \mathbf{9} = \mathbf{18}$.
 - Then $\mathbf{18} + \mathbf{18} = \mathbf{36}$.

Number 9

Properties

- The number 9 is odd.
- When multiplying by 9, the digits of the product add up to **9**.
 - $9 \times 7 = 63$. Then, $6 + 3 = \mathbf{9}$.
- For the products of 9×1 through 9×10 :
 - The **left-hand digit** of the product increases by one: **09**, **18**, **27**, **36**, **45**, **54**, **63**, etc.
 - The **right-hand digit** of the product decreases by one: **09**, **18**, **27**, **36**, **45**, **54**, **63**, etc.

Techniques

- When multiplying by 9, multiply with **10** instead. Then subtract the **other number** from the product to get the answer.
 - $9 \times \mathbf{8}$, so $\mathbf{10} \times \mathbf{8} = 80$. Then $80 - \mathbf{8} = \mathbf{72}$.
- When multiplying by 9, subtract 1 from the other number and it will become the **left-hand digit** of the product.
 - 9×5 , so $5 - 1 = \mathbf{4}$. We have $9 \times 5 = \mathbf{4}$ __.
 - To get the **right-hand digit**, recall from above that the sum of the digits of that product must equal 9.
 - So, $\mathbf{4} + \mathbf{?} = 9$. And that is **5**. Thus, $9 \times 5 = \mathbf{45}$.

Number 6

Properties

- The number 6 is even.
- When multiplying by 6, then adding digits of the product, the sum alternates from **6**, **3**, and **9**.
 - $6 \times 1 = 6$.
 - $6 \times 2 = 12$ then $1 + 2 = 3$.
 - $6 \times 3 = 18$ then $1 + 8 = 9$.
 - $6 \times 4 = 24$ then $2 + 4 = 6$, etc.

Techniques

- When 6 is multiplied by an *even* number (from 2-8), that even number will match the **right-hand digit** of the product. The **left-hand digit** of the product will be **half** of the **right-hand digit**.
 - $6 \times 2 = 12$.
 - $6 \times 4 = 24$.
 - $6 \times 6 = 36$.
 - $6 \times 8 = 48$.

Number 7

Properties

- The number 7 is odd.

Techniques

- For 7×3 , think **7**, **3**, **2**, **1** or $7 \times 3 = 21$.
- For 7×6 , think **7**, **6**, **4**, **2** or $7 \times 6 = 42$. (After 6, goes down by *even* numbers)
- For 7×8 , think **5**, **6**, **7**, **8** or $56 = 7 \times 8$.
- If you watch NFL football, you know that a touchdown is 7 points. Use the multiples of a touchdown to help you with the 7 times table.
 - Ex: $1 \times 7 = 7$, $2 \times 7 = 14$, $3 \times 7 = 21$, $4 \times 7 = 28$, $5 \times 7 = 35$, $6 \times 7 = 42$, $7 \times 7 = 49$, etc.

Number 8

Properties

- The number 8 is even.
- When multiplying by 8, then adding digits of the product, sum starts at 8 then **decreases** by one.
 - $8 \times 1 = 8$.
 - $8 \times 2 = 16$ then $1 + 6 = 7$.
 - $8 \times 3 = 24$ then $2 + 4 = 6$.

Techniques

- To multiply by 8: double, double, double. A triple-double.
 - For 8×6 :
 - Double **6** is **12**.
 - Double **12** is **24**.
 - Double **24** is **48**.

Number 12

Properties

- The number 12 is even.
- When multiplying by 3, then adding digits of the product, the sum alternates from **3**, **6**, and **9**.
 - $12 \times 1 = 12$ then $1 + 2 = \mathbf{3}$.
 - $12 \times 2 = 24$ then $2 + 4 = \mathbf{6}$.
 - $12 \times 3 = 36$ then $3 + 6 = \mathbf{9}$.

Techniques

- When multiplying by 12 (Ex: 12×4):
 - Multiply the other number by **10**. So, $4 \times \mathbf{10} = \mathbf{40}$.
 - Multiply the other number by **2**. Then, $4 \times \mathbf{2} = \mathbf{8}$.
 - Then add the two products for the answer. Finally, $\mathbf{40} + \mathbf{8} = \mathbf{48}$.

Summary

- The *Grid Format* multiplication table is a listing of products that were already calculated for you.
 - You do not have to do any computation.
- When studying the *Grid Format* multiplication table:
 - You need to know only half of the table.
 - Be mindful to the patterns of each number.
 - Highlight the grid's cells for difficult to remember facts.
 - Secure "beachheads" in each row by memorizing easy products. Then add or subtract multiples of that number to arrive at the answer.
- You must link the question (factors) to the answer (product).
 - "The factors of 5 and 9 result in what product?" Or "5 times 9 is what?"
- And you must link the answer (product) to the question (factors).
 - "The product of 45 has which factors?" Or "What number times what number is 45?"
- You must be able to work in both directions:
 - Given two factors, recall the product.
 - Given the product, recall **all** the factors. A product may have multiple factor combinations.
 - "What are the factors of 24?" Besides $\mathbf{1} \times \mathbf{24}$, there is $\mathbf{2} \times \mathbf{12}$, $\mathbf{3} \times \mathbf{8}$, and $\mathbf{4} \times \mathbf{6}$.
- When recalling the multiplication facts, my mind *does not* actually crunch the numbers.
- Recalling the facts can be viewed as attaching a *label* to a facts question, then recalling that label.
 - For example, recalling your zip code, model year of your car, P Number, etc.

References

- [Multiplication Facts Learning Strategies](https://www.youtube.com/watch?v=zlOqlqetWv8) (youtube.com/watch?v=zlOqlqetWv8), by *George Hartas*.
- [Multiplication Tips and Tricks](https://mathsisfun.com/multiplication-tips-tricks.html) (mathsisfun.com/multiplication-tips-tricks.html), by *MathsIsFun.com*.