Reduce Fraction to Lowest Terms

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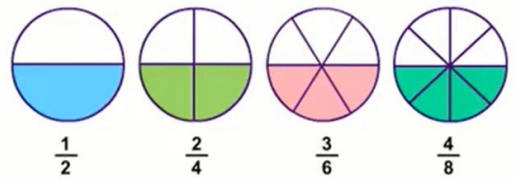
Lecture Notes

Notes

- A fraction is a **ratio** of two numbers: **numerator** with respect to **denominator**.
  - A ratio shows the relative size of two values.
  - Two quantities are compared with division, in fraction form.
  - Ex: $\frac{Numerator}{Denominator}$ or $\frac{Part}{Whole}$ or $\frac{2}{3}$ or $\frac{8}{12}$
- To **simplify** means to **reduce** a fraction to its **lowest terms** (lowest number).
  - When reducing a fraction, *both* the numerator *and* denominator are reduced.
  - The numerator and denominator are made smaller.
- **Caution:** reduce a fraction *only* if it is possible to do so.
  - If you are trying to reduce a fraction but you find that the fraction will not reduce, that means the fraction is already simplified.
  - You cannot reduce a fraction that is already reduced.
  - You can only reduce if you have the same number in the numerator and denominator. If the same number appears only in the numerator, you cannot cancel them. The same applies for the denominator. You *cannot cancel the same number side-to-side*, only up and down.

Equivalent Fractions

- **Equivalent fractions** represent the same *value*, although they look different.
- If you *multiply* or *divide* both the numerator and the denominator by the **same number**, you will get an **equivalent fraction**.
  - *Multiply* by 2: $Start\ with\ \frac{1}{2}\ \ then\ \frac{1\cdot 2}{2\cdot 2}=\frac{2}{4}\ \ then\ \frac{2\cdot 2}{4\cdot 2}=\frac{4}{8}$
  - *Divide* by 2: $Start\ with\ \frac{4}{8}\ \ then\ \frac{4\div 2}{8\div 2}=\frac{2}{4}\ \ then\ \frac{2\div 2}{4\div 2}=\frac{1}{2}$
- The four fractions below have the same shaded area, one-half of the circle. The difference is that the "slices" have different sizes.

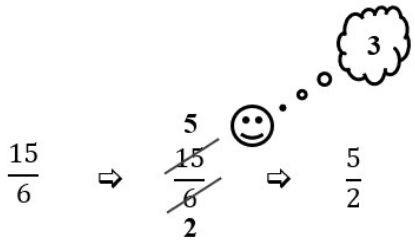


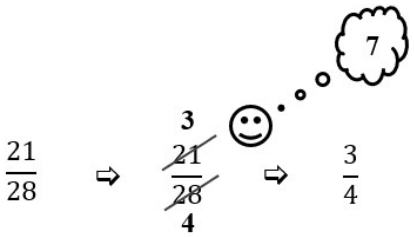
- Therefore, the three fractions on the right are equivalent to **one-half**.
  - $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$
- **Reference:** Equivalent Fractions, by *The Learning Portal*.

Option 1: Reduce by Dividing

- **Step 1: Think Big.** Think to yourself, "What is the **biggest** number that divides evenly into **both** the numerator and denominator?" Put that number in a "thought bubble." See example below.
- **Step 2: Divide.** Divide the number you are thinking of into **both** the numerator and denominator.
- **Step 3: Slash.** Slash out the old numerator and denominator.
- **Step 4: Replace.** Replace old numerator and denominator with new numerator and denominator.
- *Notes:*
  - You are finding the *Greatest Common Factor (GCF)* between numerator and denominator.
  - An alternative to using the "thought bubble" is to divide both numerator and denominator by the **GCF**. The result is the same. For example: $\frac{8}{16}\ \frac{\div 8}{\div 8} = \frac{1}{2}$
  - This method requires that you are *excellent* at recalling the multiplication facts.
  - Use this technique if the numbers are reasonable: they are in the multiplication table.
  - If the numbers are big and are difficult to reduce (they are not in the multiplication table), use *Option 2: Reduce by Prime Factorization* discussed next.

Reduce by Dividing Examples:

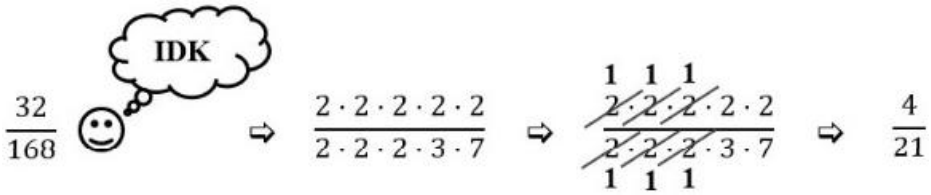




Option 2: Reduce by Prime Factorization

- **Step 1: Factor Tree.** Use the *Factor Tree* to obtain the *prime factorization* of both the numerator and denominator.
- **Step 2: Put Primes in Fraction.** Write the prime factorizations you obtained from the Factor Tree onto the numerator and denominator.
- **Step 3: Slash.** Slash out the *same* factors from the numerator *and* denominator.
  - The top and bottom numbers must be the *same* to slash ("cancel") them.
- **Step 4: Replace.** Replace the slashed out numbers in the numerator and denominator with 1.
- *Notes:*
  - For numbers to cancel, they do not have to be directly over each other.
    - The number in the numerator can be in the front of the prime factorization and the number in the denominator can be in the back. The numbers will still be able to cancel, thanks to the *Commutative Property of Multiplication*.
    - However, the numbers in both numerator and denominator must be *factors* (multiplying with other numbers next to them) to allow cancelling.
  - The reason the slashed out numbers are replaced with 1 is because a fraction means division. Dividing a number in the numerator by the same number in denominator results in 1. When two numbers are cancelled, they do not disappear. Instead, they become 1.
  - If the numbers are big and are difficult to reduce (they are not in the multiplication table), use this method. See example below.

Reduce by Prime Factorization Example:



Write the fraction in lowest terms. $\frac{42}{48}$	$\frac{42}{48} = \frac{7}{8}$ (Simplify your answer. Type a whole number or a fraction.)

- The *biggest* number that divides into *both* the 42 and 48 is 6.
- Divide 6 into *both* the numerator and denominator.
- *Slash* out old numerator and denominator.
- *Replace* the numerator (42 with **7**) and denominator (48 with **8**).

Write the fraction in lowest terms. $\frac{5}{15}$	$\frac{5}{15} = \frac{1}{3}$ (Simplify your answer. Type a whole number or a fraction.)

- The *biggest* number that divides into *both* the 5 and 15 is 5.
- Divide 5 into *both* the numerator and denominator.
- *Slash* out old numerator and denominator.
- *Replace* the numerator (5 with **1**) and denominator (15 with **3**).
- Note: If a fraction reduces where all the factors in the numerator "cancel" out, a 1 must be placed in the numerator to act as a placeholder. The numerator cannot be left blank.

Write the fraction in lowest terms. $\frac{232}{116}$	$\frac{232}{116} = 2$ (Simplify your answer. Type a whole number or a fraction.)

- If numerator is bigger than denominator, see if the denominator divides evenly into the numerator.
- In this problem, it does. Exactly **2** times.
- This fraction is considered to be reduced.

Write the fraction in lowest terms. $\frac{49}{63}$	$\frac{49}{63} = \frac{7}{9}$ (Simplify your answer. Type an integer or a fraction.)

- The *biggest* number that divides into *both* the 49 and 63 is 7.
- Divide 7 into *both* the numerator and denominator.
- *Slash* out old numerator and denominator.
- *Replace* the numerator (49 with **7**) and denominator (63 with **9**).

Find the correct prime factorization of $\frac{45}{75}$, and then reduce the fraction to lowest terms.

Choose the correct prime factorization below.

- A. $\frac{3 \cdot 5 \cdot 5}{3 \cdot 3 \cdot 5 \cdot 5}$
- B. $\frac{3 \cdot 15}{3 \cdot 25}$
- C. $\frac{3 \cdot 3 \cdot 5 \cdot 45}{3 \cdot 5 \cdot 5 \cdot 75}$
- D. $\frac{3 \cdot 3 \cdot 5}{3 \cdot 5 \cdot 5}$

$\frac{45}{75}$ in lowest terms is $\frac{3}{5}$.

Find the prime factorization of the numerator and the denominator. Then write the fraction in lowest terms.

$\frac{125}{175}$

Choose the correct prime factorization of $\frac{125}{175}$.

- A. $\frac{5 \cdot 125}{5 \cdot 175}$
- B. $\frac{5 \cdot 25}{5 \cdot 35}$
- C. $\frac{5 \cdot 5 \cdot 5 \cdot 7}{5 \cdot 5 \cdot 7 \cdot 7}$
- D. $\frac{5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 7}$

Write $\frac{125}{175}$ in lowest terms. $\frac{5}{7}$

Write the numerator and denominator as a product of prime factors. Then write the fraction in lowest terms.

$\frac{10}{35}$

Choose the correct prime factorizations.

- A. $\frac{5 \cdot 35}{5 \cdot 49}$
- B. $\frac{2 \cdot 5}{7 \cdot 5}$
- C. $\frac{5 \cdot 7 \cdot 7}{5 \cdot 5 \cdot 7 \cdot 7}$
- D. $\frac{5 \cdot 5 \cdot 7 \cdot 10}{5 \cdot 7 \cdot 7 \cdot 35}$

$\frac{10}{35}$ in lowest terms is $\frac{2}{7}$.