

# DERIVATIVES AND INTEGRALS

## Basic Differentiation Rules

1.  $\frac{d}{dx}[cu] = cu'$
2.  $\frac{d}{dx}[u \pm v] = u' \pm v'$
3.  $\frac{d}{dx}[uv] = uv' + vu'$
4.  $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5.  $\frac{d}{dx}[c] = 0$
6.  $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7.  $\frac{d}{dx}[x] = 1$
8.  $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9.  $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10.  $\frac{d}{dx}[e^u] = e^u u'$
11.  $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12.  $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13.  $\frac{d}{dx}[\sin u] = (\cos u)u'$
14.  $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15.  $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16.  $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17.  $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18.  $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19.  $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20.  $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21.  $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22.  $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$
23.  $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24.  $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25.  $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26.  $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27.  $\frac{d}{dx}[\tanh u] = (\sech^2 u)u'$
28.  $\frac{d}{dx}[\coth u] = -(\csch^2 u)u'$
29.  $\frac{d}{dx}[\sech u] = -(\sech u \tanh u)u'$
30.  $\frac{d}{dx}[\csch u] = -(\csch u \coth u)u'$
31.  $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$
32.  $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$
33.  $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$
34.  $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$
35.  $\frac{d}{dx}[\sech^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$
36.  $\frac{d}{dx}[\csch^{-1} u] = \frac{-u'}{|u|\sqrt{1+u^2}}$

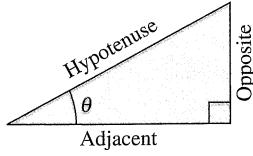
## Basic Integration Formulas

1.  $\int kf(u) du = k \int f(u) du$
2.  $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$
3.  $\int du = u + C$
4.  $\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$
5.  $\int e^u du = e^u + C$
6.  $\int \sin u du = -\cos u + C$
7.  $\int \cos u du = \sin u + C$
8.  $\int \tan u du = -\ln|\cos u| + C$
9.  $\int \cot u du = \ln|\sin u| + C$
10.  $\int \sec u du = \ln|\sec u + \tan u| + C$
11.  $\int \csc u du = -\ln|\csc u + \cot u| + C$
12.  $\int \sec^2 u du = \tan u + C$
13.  $\int \csc^2 u du = -\cot u + C$
14.  $\int \sec u \tan u du = \sec u + C$
15.  $\int \csc u \cot u du = -\csc u + C$
16.  $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$
17.  $\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
18.  $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

# TRIGONOMETRY

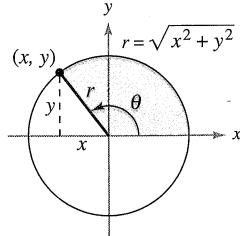
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$ .

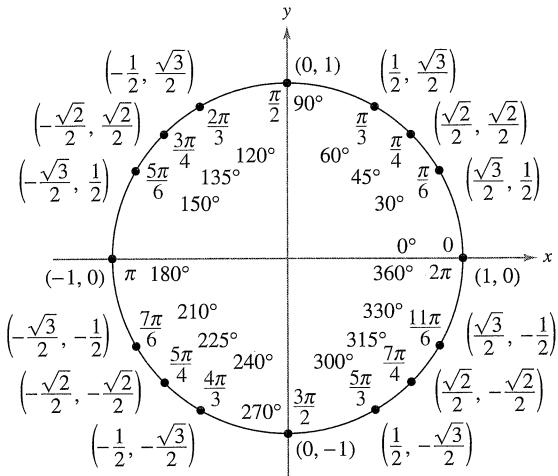


$$\begin{array}{ll} \sin \theta = \frac{\text{opp}}{\text{hyp}} & \csc \theta = \frac{\text{hyp}}{\text{opp}} \\ \cos \theta = \frac{\text{adj}}{\text{hyp}} & \sec \theta = \frac{\text{hyp}}{\text{adj}} \\ \tan \theta = \frac{\text{opp}}{\text{adj}} & \cot \theta = \frac{\text{adj}}{\text{opp}} \end{array}$$

Circular function definitions, where  $\theta$  is any angle.



$$\begin{array}{ll} \sin \theta = \frac{y}{r} & \csc \theta = \frac{r}{y} \\ \cos \theta = \frac{x}{r} & \sec \theta = \frac{r}{x} \\ \tan \theta = \frac{y}{x} & \cot \theta = \frac{x}{y} \end{array}$$



## Reciprocal Identities

$$\begin{array}{lll} \sin x = \frac{1}{\csc x} & \sec x = \frac{1}{\cos x} & \tan x = \frac{1}{\cot x} \\ \csc x = \frac{1}{\sin x} & \cos x = \frac{1}{\sec x} & \cot x = \frac{1}{\tan x} \end{array}$$

## Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

## Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \quad 1 + \cot^2 x = \csc^2 x \end{aligned}$$

## Cofunction Identities

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

## Reduction Formulas

$$\begin{aligned} \sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x \end{aligned}$$

## Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$

## Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

## Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

## Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

## Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$