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## Domain Issues of Inverse Composite Trig Functions

Case 1: Inner Function is the Restricted Function

- If argument ( $\theta$ ) is not within the domain, use its reference angle instead. For cos, it is easier to apply odd/even property.

Restricted Function Domain and Range:

$$y = \sin(\theta) \quad D: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$R: [-1, 1]$$

$$y = \cos(\theta) \quad D: [0, \pi]$$

$$R: [-1, 1]$$

$$y = \tan(\theta) \quad D: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \leftarrow \text{Note use of } ()$$

$$R: (-\infty, \infty)$$

- Then,

$$\sin^{-1}(\sin(\theta)) = \theta$$

$$\cos^{-1}(\cos(\theta)) = \theta$$

$$\tan^{-1}(\tan(\theta)) = \theta$$

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Case 2: Inner Function is the Inverse Function

- If argument (trig. value) is not within the domain, then the outer (restricted) function is also not defined.

Inverse Function Domain and Range:

$$y = \sin^{-1}(x) \rightarrow \theta = \sin^{-1}(x)$$

$$D: [-1, 1]$$

$$R: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y = \cos^{-1}(x) \rightarrow \theta = \cos^{-1}(x)$$

$$D: [-1, 1]$$

$$R: [0, \pi]$$

$$y = \tan^{-1}(x) \rightarrow \theta = \tan^{-1}(x)$$

$$D: (-\infty, \infty)$$

$$R: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \leftarrow \text{Note use of } (\ )$$

- Then, the composite function is not defined.
- No real solution.
- If defined, then

$$\left. \begin{aligned} \sin(\sin^{-1}(x)) &= x \\ \cos(\cos^{-1}(x)) &= x \\ \tan(\tan^{-1}(x)) &= x \end{aligned} \right\} \begin{array}{l} x \\ \text{mean } s \\ \text{trig value} \end{array}$$