Reciprocal Property for Inequalities

In the Sullivan *Algebra & Trigonometry* textbook, this property is stated as:

Reciprocal Property for Inequalities

If $a > 0$, then $\frac{1}{a} > 0$ If a number is positive, then its reciprocal is also positive.

If $a < 0$, then $\frac{1}{a} < 0$ If a number is negative, then its reciprocal is also negative.

Case 1: If $a > 0$, then $\frac{1}{a} > 0$

Example:

$(3x - 12)^{-1} > 0$

$\frac{1}{3x - 12} > 0$

$3x - 12 > 0$

$3x > 12$

$x > 4$

My Thoughts on the *Reciprocal Property for Inequalities*

The principle is a shortcut for simplifying a rational inequality in the form of $\frac{1}{a} > 0$ or $\frac{1}{a} < 0$.

It was difficult to understand the shortcut jump made from $\frac{1}{3x-12} > 0$ to $3x - 12 > 0$.

I needed to see the intermediate steps of the shortcut so I can understand how this principle is valid. So instead of using the *Reciprocal Property for Inequalities*, I used algebraic techniques to solve an example rational inequality.

Solving an Example Using Algebraic Techniques:

$(3x-12)^{-1} > 0$

$(3x-12)^{+2} * (3x-12)^{-1} > 0 * (3x-12)^{+2}$ Multiply both sides by exponent $^{+2}$. Add exponents.

$(3x-12)^{+1} > 0$ Result on left side has an exponent $^{+1}$. Right side value remains 0.

$3x-12 > 0$ An expression with exponent $^{+1}$ is that same expression; removed parenthesis.

$3x > 12$

$x > 4$

Notes:

- Since exponent of original problem is -1, I multiplied both sides by the same expression but with an exponent of $+2$. I chose $+2$ because when multiplying the binomials on the left, the exponents would sum up to $+1$ and therefore I can remove the parenthesis and solve for x.
- This technique also works for the case:
  - If $a < 0$, then $\frac{1}{a} < 0$
  - Because 0 multiplied by a squared power is still zero, the inequality remains true.

A Simpler Example Using Algebraic Techniques:

Case 2: If $a < 0$, then $\frac{1}{a} < 0$

Let $a = -2$

$-2 < 0$

$-2 * \left(-\frac{1}{2}\right)^2 < 0 * \left(-\frac{1}{2}\right)^2$ Multiply both sides by $\left(-\frac{1}{2}\right)^2$.

$-2 * \left(\frac{1}{4}\right) < 0$

$-\frac{1}{2} < 0$ Inequality is still true.