Section 2.2 - Using Second Derivative to Find Max/Min Values & Sketch Graph

Steps A-F below are kept the same as the textbook (page 222).

Last Updated: 3/20/15

This Guide Sheet is for solving **only** this specific question (Exercise Set 2.2, problems 9-46): *"Sketch the graph. List the coordinates of where extrema or points of inflection occur. State where the function is increasing or decreasing, as well as where it is concave up or concave down."*

Original Function: $f(x) = 3x^5 - 20x^3$ ← This is Example 2 and 3 on pages 219-222 of textbook.

Step A – Find derivatives and domain.

> **Step A.1** – Find $f'(x)$.
>
> $f'(x) = 15x^4 - 60x^2$

> **Step A.2** – Find $f''(x)$.
>
> $f''(x) = 60x^3 - 120x$

> **Step A.3** – Find domain. No DNEs.
>
> ***Note:*** Section 2.2 has mainly polynomial functions so domain is all real numbers. There is no x that does not exist (DNE) for these type of functions.

Step B – Find *critical values* and *critical points.*

> **Step B.1** – Find *critical values* (x-coordinate) by solving for $f'(x) = 0$ and where $f'(x)$ = DNE. Set function equal to zero, factor, then use the *Principle of Zero Products.*
>
> $15x^4 - 60x^2 = 0$
>
> $15x^2(x^2 - 4) = 0$
>
> $15x^2(x+2)(x-2) = 0$
>
> $15x^2 = 0$, $x+2=0$, $x-2=0$
>
> $x = 0$, $x = -2$, $x = 2$
>
> ***Note:*** Because the domain of $f(x)$ is all real numbers, there are no DNEs.

> **Step B.2** – Find corresponding y-coordinate by substituting each *critical value* (*cv*) into $f(x)$.
>
> $f(-2) = 3(-2)^5 - 20(-2)^3 = 64$
>
> $f(0) = 3(0)^5 - 20(0)^3 = 0$
>
> $f(2) = 3(2)^5 - 20(2)^3 = -64$

> **Step B.3** – List *critical points* (x, y coordinates).
>
> $(-2, 64)$ $(0, 0)$ $(2, -64)$ (,)