Matrices

Business Precalculus, 10th Ed. 2012

2.2 - Solution of Linear Systems by the Gauss - Jordan Method

Row Operations

1) Interchange any two rows
2) Multiply row by any nonzero number
3) Multiply one row and add it to another row.

Gauss - Jordan Method

1) Put variables in same order on left of equals sign and constants on right side.
2) Write coefficients (and signs) in matrix.
3) Work one column at a time from left to right.
4) First get 0's above and below the main diagonal using row operations. Multiply by a ± fraction if needed.
5) If a diagonal number is not a 1, multiply by that number's reciprocal so it becomes 1.

Sample Solution

Diagonal one's

$$\begin{array}{cc} x & \\ \downarrow & \\ \left[\begin{array}{cc|c} \boxed{1} & 0 & -2 \\ 0 & \boxed{1} & 3 \end{array}\right] \\ & \uparrow \\ & y \end{array}$$

so $x = -2$
$y = 3$

Sample No Solution

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & -1 \end{array}\right] \leftarrow 0 + 0 \neq -1$$

Sample Infinite Solutions

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{7} & \frac{12}{7} \\ 0 & 1 & -\frac{4}{7} & -\frac{6}{7} \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow 0 = 0$$

- Infinite solutions for z; infinite values for z.
- Write equations that correspond to first 2 rows of matrix.

Ex:

$$x + \frac{1}{7}z = \frac{12}{7}$$

$$y - \frac{4}{7}z = -\frac{6}{7}$$

- Solve 2 equations for x and y:

$$x = \frac{12 - z}{7} \quad \text{and} \quad y = \frac{4z - 6}{7}$$

- Write answer as a General Solution:

$$\left(\underset{x}{\frac{12-z}{7}}, \underset{y}{\frac{4z-6}{7}}, \underset{z}{z} \right)$$

z can be any number.

2.3 Addition and Subtraction of Matrices

- To add or subtract matrices, they must be equal size (same number of rows and columns).
- Add or subtract each corresponding element of the two matrices into a third (new) matrix of the same size.
- When subtracting, watch for the '−' sign.

2.4 Multiplication of Matrices

Product of Scalar (real number) and Matrix

- Multiply scalar with each element in matrix and put result in a new matrix of the same size.

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Product of Two Matrices

1) Matrix multiplication is not commutative so multiplying matrices in the correct order is important.

$$AB \neq BA$$

2) Check row/column size of matrices A and B.

a) The number of columns of A must equal the number of rows of B.

b) If matrix A is $m \times n$ size (rows x columns) then matrix B <u>must</u> be $n \times k$ size to multiply.

Ex: $A = \begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$

Size of A is 2×3, size of B is 3×2.

The 2 inside numbers match so ok to multiply.

c) The size of the product matrix (output matrix) is $m \times k$.

Ex: Based on example above, see that the product matrix is 2×2, which are the 2 outside numbers.

$$AB = \begin{bmatrix} 32 & -4 \\ -18 & 12 \end{bmatrix}$$

3) Begin by multiplying each element of Row 1 of matrix A (A:R1) with each corresponding element of Column 1 of matrix B (B:C1).

A:R1 and B:C1 each have 3 elements.

Sum up the 3 products and put result into Row 1 Column 1 (AB:R1C1) of the product matrix.

Ex:

Row 1 Column 1

$$A \rightarrow \begin{bmatrix} -3 & 4 & 2 \\ & & \end{bmatrix} \begin{bmatrix} -6 \\ 2 \\ 3 \end{bmatrix} \leftarrow B$$

$$(-3)\cdot(-6) + 4\cdot 2 + 2\cdot 3 = 32$$

The result of multiplying Row 1 • Column 1 goes into the <u>same spot</u> (R1C1) in product matrix.

$$AB = \begin{bmatrix} 32 & \\ & \end{bmatrix}$$

4) Follow the same procedure as in step 3 above to multiply the rest of matrix A and B.

A:R1 • B:C2
A:R2 • B:C1
A:R2 • B:C2

5) The completed product matrix should look like this.

$$\begin{array}{ccc} & R1C1 & R1C2 \\ AB = & \begin{bmatrix} 32 & -4 \\ -18 & 12 \end{bmatrix} & \\ & R2C1 & R2C2 \end{array}$$