Least Common Multiple

Definition:

- **Least common multiple (LCM)** is the *smallest* number that is a multiple of two numbers. The LCM can be used to find the **least common denominator (LCD)** of fractions.

Here are four methods for finding the LCM. You will most often use **Method B** and **Method C**. Method D is useful for finding the LCM for three or more numbers. It is highly recommended to learn Method C at a minimum.

METHOD A: Find LCM by listing multiples of each number.

Steps:

1. List multiples of both numbers until you find the *first* multiple that is included in both lists. That is the LCM.

Example: Find the LCM of 20 and 30.

List multiples of 20 by multiplying 20 by 1, 2, 3, etc.
20, 40, 60

List multiples of 30 by multiplying 30 by 1, 2, 3, etc.
30, 60

The *first* multiple that is included in both lists is 60 and so it is the LCM. The LCM of 60 is the smallest number common to both lists.

Note: The multiple 120 also matches both numbers as do other larger numbers. However, they are not the *least* common multiple because they are not the *first* match common to both numbers. Number 60 is the *lowest (least)* multiple that is common to 20 and 30.

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METHOD B: Find LCM by listing multiples of larger number only. This method is a shortcut for Method A because only one list is used.

Steps:

1. See if larger number is a multiple of the smaller number. If it is, then the larger number is the LCM.
2. If larger number is not a multiple of the smaller number, continue checking multiples of the larger number until you find the *first* multiple that is also a multiple of the smaller number.

Example: Find the LCM of 9 and 12.

Step 1: Is larger number '12' a multiple of the smaller number '9'? (Does 9 divide evenly into 12 with no remainder?) No.

Step 2: Continue by checking the next higher multiple of the larger number 12, which is 24. Is 24 a multiple of 9? No.

Repeat Step 2: Again, check the next higher multiple of the larger number 12, which is 36. Is 36 a multiple of 9? Yes.

The *first* multiple (36) of the higher number (12) which is evenly divided by the smaller number (9) is the LCM.

Here is a summary of the steps:
$1 \cdot 12 = 12$, not a multiple of 9
$2 \cdot 12 = 24$, not a multiple of 9
$3 \cdot 12 = 36$, is a multiple of 9 because $4 \cdot 9 = 36$

The LCM of 9 and 12 is 36.

METHOD C: Find LCM using prime factorization.

Steps:

1. Write the prime factorization of each number.
2. Create an *LCM factor list* and include factors occurring the *greatest* number of times in any one factorization.
   Notes:
   - » Include factors when they exist in one number but not in the other number(s).
   - » If factors are common among two (or more) numbers, do not include repeat factors.
   - » See *Prime Factorization* sheet for finding prime factors.

Example 1: Find the LCM of 24 and 36.

Step 1: Write the prime factorization of each number.
24 = 2 • 2 • 2 • 3
36 = 2 • 2 • 3 • 3

Step 2: Create an *LCM factor list* and include factors occurring the *greatest* number of times in any one factorization.

The factor 2 – occurs three times with number 24 and two times with number 36. Include the factor 2 three times on the LCM factor list below.

The factor 3 – occurs two times with number 36 and one time with number 24. Include the factor 3 two times on the LCM factor list below.

LCM factor list: 2 • 2 • 2 • 3 • 3

The LCM of 24 and 36 is 2 • 2 • 2 • 3 • 3 = 72.

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Example 2: Find LCM of 81, 84, and 90.

Step 1: Write the prime factorization of each number.
81 = 3 • 3 • 3 • 3
84 = 2 • 2 • 3 • 7
90 = 2 • 3 • 3 • 5

Step 2: Create an *LCM factor list* and include factors occurring the *greatest* number of times in any one factorization.

The factor 2 – occurs two times with number 84, one time with number 90, and is not a factor with number 81. Include the factor 2 two times on the LCM factor list below.

The factor 3 – occurs four times with number 81, two times with number 90, and one time with number 84. Include the factor 3 four times on the LCM factor list below.

The factor 5 – occurs one time with number 90, and is not a factor with numbers 81 or 84. Include the factor 5 one time on the LCM factor list below.

The factor 7 – occurs one time with number 84, and is not a factor with numbers 81 or 90. Include the factor 7 one time on the LCM factor list below.

LCM factor list: 2 • 2 • 3 • 3 • 3 • 3 • 5 • 7

The LCM of 81, 84, and 90 is 2 • 2 • 3 • 3 • 3 • 3 • 5 • 7 = 11,340.

METHOD D: Find LCM using division by primes.

This method is best for finding LCM for three or more numbers.

Steps:

1. Find a prime that divides evenly (no remainder) into any *two* of the numbers.
2. Divide and bring down the answer (quotient) for each number. Also, bring down any numbers not evenly divisible by the prime.
    a. If there is no prime that divides evenly into at least *two* of the numbers, the LCM is the product of the numbers.
3. Repeat the process until there are no *two* numbers divisible by the same prime.

Example: Find the LCM of 48, 72, and 80.

Step 1: Find a prime that divides evenly (no remainder) into any *two* of the numbers.

That prime is 2 since it divides evenly into at least *two* of the numbers.

48 72 80

Step 2: Divide and bring down the answer (quotient) for each number. Also, bring down any numbers not evenly divisible by the prime.

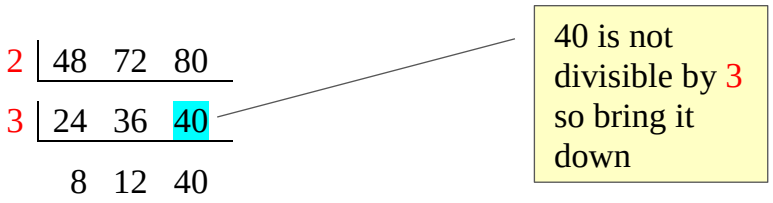
Iteration 1 – See below. The prime 2 divides evenly into at least *two* of the numbers: 48, 72, and 80. Divide 2 into 48 for a quotient of 24. Divide 2 into 72 for a quotient of 36. Divide 2 into 80 for a quotient of 40. Write the answer below each number.

2	48 72 80
	24 36 40

Continued

Step 3: Repeat the process until there are no *two* numbers divisible by the same prime.

Iteration 2 – See below. The prime 3 divides evenly into at least *two* of the numbers: 24 and 36 but not 40. Divide 3 into 24 for a quotient of 8. Divide 3 into 36 for a quotient of 12. Since 40 is not divisible by the prime 3, bring down the 40. Write the answer below each number.



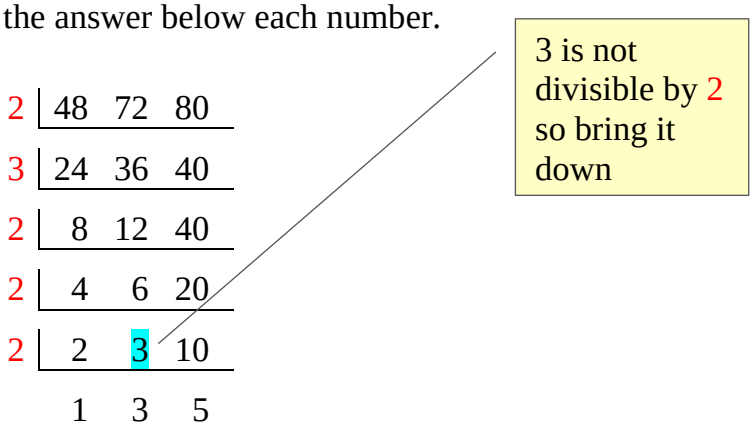
Iteration 3 – See below. The prime 2 divides evenly into at least *two* of the numbers: 8, 12, and 40. Divide 2 into 8 for a quotient of 4. Divide 2 into 12 for a quotient of 6. Divide 2 into 40 for a quotient of 20. Write the answer below each number.

2	48	72	80
3	24	36	40
2	8	12	40
	4	6	20

Iteration 4 – See below. The prime 2 divides evenly into at least *two* of the numbers: 4, 6, and 20. Divide 2 into 4 for a quotient of 2. Divide 2 into 6 for a quotient of 3. Divide 2 into 20 for a quotient of 10. Write the answer below each number.

2	48	72	80
3	24	36	40
2	8	12	40
2	4	6	20
	2	3	10

Iteration 5 – See below. The prime 2 divides evenly into at least *two* of the numbers: 2 and 10 but not 3. Divide 2 into 2 for a quotient of 1. Since 3 is not divisible by the prime 2, bring down the 3. Divide 2 into 10 for a quotient of 5. Write the answer below each number.



Iteration 6 – See below. **No** prime divides evenly into at least *two* of the numbers: 1, 3, and 5. Stop iterations here. The remaining numbers on the bottom 1, 3, and 5 become factors of the LCM.

2	48 72 80
3	24 36 40
2	8 12 40
2	4 6 20
2	2 3 10
	1 3 5

See above. The LCM factor list is: 2 • 3 • 2 • 2 • 2 • 1 • 3 • 5

Therefore, the LCM of 48, 72, and 80 is 2 • 3 • 2 • 2 • 2 • 1 • 3 • 5 = 720.