Add Fractions

Example: Add and simplify $\frac{7}{24} + \frac{15}{36}$

STEP 1 – LCD: To find the **L**owest **C**ommon **D**enominator (**LCD**), we can use either of the two methods for finding the Least Common Multiple (LCM). Examine denominators to determine which method to use. Perhaps 24 and 36 will have a match within the first few multiples. Let's try *List of Multiples* method first. If after the first few multiples we see that 24 and 36 do not have a common multiple, then we can switch to *Shares & Leftovers* method. We write multiples of 24 and 36 and notice that the LCD = 72.

24	36
48	**72**
72	

List multiples of **24** and **36**.

STEP 2 – Multiply by $\frac{n}{n}$: Multiply numerator *and* denominator ($\frac{n}{n}$) of each fraction that does not originally have the LCD (72) as its denominator. Both denominators are not 72. Thus, we multiply both fractions by some number to make them become the LCD (72). Start with the left fraction. Place a multiplication dot " • " to the left of 24. Ask yourself, "What number times 24 is 72?" Use trial and error to get 3 • 24 = 72. Thus, multiply both denominator *and* numerator by 3. Using the same procedure for the right fraction, we find 2 • 36 = 72. Multiply both denominator *and* numerator by 2.

$$\frac{3 \cdot 7}{3 \cdot 24} + \frac{2 \cdot 15}{2 \cdot 36} \Rightarrow \frac{21}{72} + \frac{30}{72}$$

Multiply numerator and denominator by the same number ($\frac{n}{n}$).

Note: The number we multiply each fraction by will be the same "up and down" because we must maintain the ratio of numerator *to* denominator. However, that number will be different between the two fractions because the original denominators are different.

STEP 3 – Add: We add numerators and keep denominator the same to get $\frac{51}{72}$.

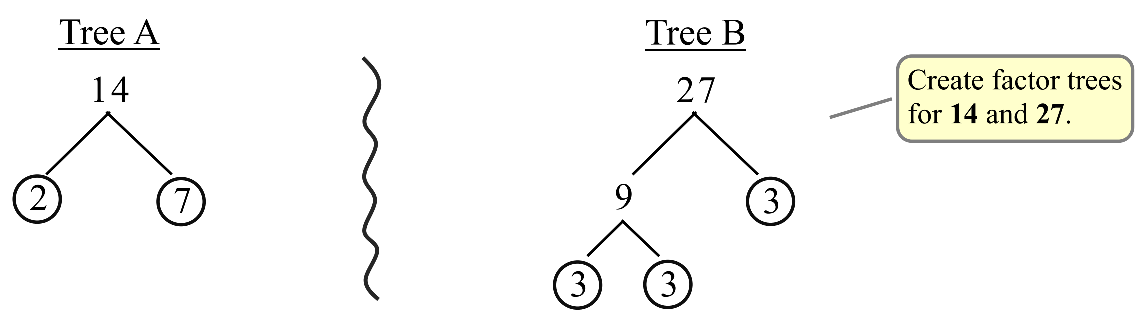
STEP 4 – Reduce: If possible, reduce $\frac{51}{72}$. Both numerator and denominator are big numbers making it difficult to know if this fraction reduces. However, using our knowledge of *divisibility rules,* we find that 51 and 72 are both divisible by 3. Thus, we divide numerator and denominator by 3 to arrive at the answer: $\frac{17}{24}$.

Subtract Fractions

Example: Subtract and simplify $\dfrac{8}{14} - \dfrac{9}{27}$

The process of subtracting fractions is identical to adding fractions. Of course, the exception is that in Step 3, we subtract numerators instead of adding them.

STEP 1 – LCD: Examining the denominators, 14 and 27, it seems that we will not get a quick match (within the first few multiples) by using the *List of Multiples* method to find the LCD. We will try the *Shares & Leftovers* method instead. Complete a *factor tree* for both 14 and 27 to obtain their *prime factorization*.



List the prime factorization for 14 and 27 in a horizontal row from smallest to largest. Use the *Shares & Leftovers* technique to determine which factors to include as part of the LCM. There are no *Shares* but instead, all the factors are *Leftovers*. Therefore, bring down *all* factors from both rows.

14: 2 • 7
27: 3 • 3 • 3
LCM: 2 • 7 • 3 • 3 • 3 = 378

> There are no **Shares** factors. All factors are **Leftovers**. Multiply all Leftover factors for the LCM.

STEP 2 – Multiply by $\dfrac{n}{n}$: Multiply numerator *and* denominator ($\dfrac{n}{n}$) of each fraction that does not originally have the LCD (378) as its denominator. Both denominators are not 378. Thus, we multiply both fractions by some number to make them become the LCD (378).

Left Fraction (Next page): Place a multiplication dot " • " to the left of 14. Ask yourself, "What number times 14 is 378?" This is difficult with trial and error. However, we will use the *prime factorization* of the LCM from Step 1 to help us. We have 14 in the original denominator. We are looking for other factors that when multiplied by 14 will result in 378. Notice that we also have the 2 • 7 = 14 in the prime factorization 2 • 7 • 3 • 3 • 3. The remaining factors that result in 378 are 3 • 3 • 3 = 27. Thus, we multiply the 14 by 27 to get 378 in the denominator. But we also multiply the numerator by 27 to maintain the fraction's ratio.

Right Fraction: Place a multiplication dot " • " to the left of 27. Ask yourself, "What number times 27 is 378?" Again, we will use the *prime factorization* of the LCM from Step 1 to help us. We have 27 in the original denominator. We are looking for other factors that when multiplied by 27 will result in 378. Notice that we also have the 3 • 3 • 3 = 27 in the prime factorization 2 • 7 • 3 • 3 • 3. Observe that the remaining factors resulting in 378 are 2 • 7 = 14. Thus, we multiply the 27 by 14 to get 378 in the denominator. And we also multiply the numerator by 14 to maintain the fraction's ratio.

$$\frac{27 \cdot 8}{27 \cdot 14} - \frac{14 \cdot 9}{14 \cdot 27} \Rightarrow \frac{216}{378} - \frac{126}{378}$$

Multiply numerator and denominator by the same number ($\frac{n}{n}$).

STEP 3 – Subtract: We subtract numerators and keep denominator the same to get $\frac{90}{378}$.

STEP 4 – Reduce: If possible, reduce $\frac{90}{378}$. Both numerator and denominator are big numbers making reducing difficult. But using our knowledge of *divisibility rules,* we see that 90 and 378 are *even* numbers so they are divisible by 2. Use long division to reduce.

$$\frac{90 \div 2}{378 \div 2} \Rightarrow \frac{45}{189}$$

Divide by the common factor **2** to reduce.

Is $\frac{45}{189}$ the final answer? Use *divisibility rules* again to see. Are 45 and 189 divisible by 2 again? No since both are not even numbers. Are 45 and 189 divisible by 5? No because the ones place value digit of 45 and 189 are not both 0 or 5. Are 45 and 189 divisible by 10? No because the ones place value digit of 45 and 189 are not both 0. We check 3 or 9. For 3, add digits of numerator (4 + 5 = 9) and denominator (1 + 8 + 9 = 18). We see that the sum of the digits of numerator (9) and denominator (18) is divisible by 3. That means that the original numerator (45) and denominator (189) are also divisible by 3. Thus, we can divide numerator and denominator by 3 to reduce.

But let's also check divisibility for 9. Recall that 9 has the same divisibility rule as 3. Observe that 9 can be used to reduce as well. We have a choice to divide either by 3 or 9. We choose to divide by 9 because it will result in a greater reduction.

$$\frac{45 \div 9}{189 \div 9} \Rightarrow \frac{5}{21}$$

Divide by the common factor **9** to reduce.

Is $\frac{5}{21}$ the final answer? We use divisibility rules once again to check. We notice that the fraction does not reduce further. Additionally, our knowledge of the multiplication facts confirms there is no common factor (except 1) between 5 and 21. We are done reducing.