

## The Box Method for Factoring a Trinomial

Last Updated: 2/7/14

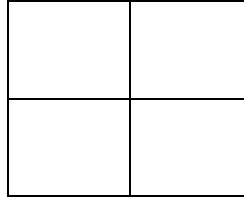
**CASE 1:** Middle term is '+' and last term is '+'.

$$7x^2 + 37x + 10$$

**Step 1:** Factor out any Greatest Common Factors (GCF). None here.

**Step 2:** Ensure a '+' leading coefficient. Factor out '-1' if needed. None here.

**Step 3:** Draw a four-square box.



**Step 4:** Write leading term  $7x^2$  in top-left box. Write last term 10 in bottom-right box.

$7x^2$	
	10

**Step 5:** Multiply leading coefficient 7 with last term 10 to get  $7 \cdot 10 = 70$ .

**Step 6:** List all factors of 70 starting with  $1 \cdot 70$  and sequentially work up from 1; see table below.

To determine the signs of the two factors we are seeking in the table, look at the sign of the last term '10' of the trinomial. That sign is '+' which means that both factors have the same sign, either both being '+' or both being '-'. To find which sign, look at the sign of the middle term  $37x$  of the trinomial. That sign is '+' which means that both factors we are seeking are also '+'. Therefore there is no need to list negative factors of 70 in the table, such as  $-1 \cdot -70$ ,  $-2 \cdot -35$ , etc.

<u>Factors of 70</u>	<u>Sum</u>
$1 \cdot 70$	71
$2 \cdot 35$	37
$5 \cdot 14$	19
$7 \cdot 10$	17

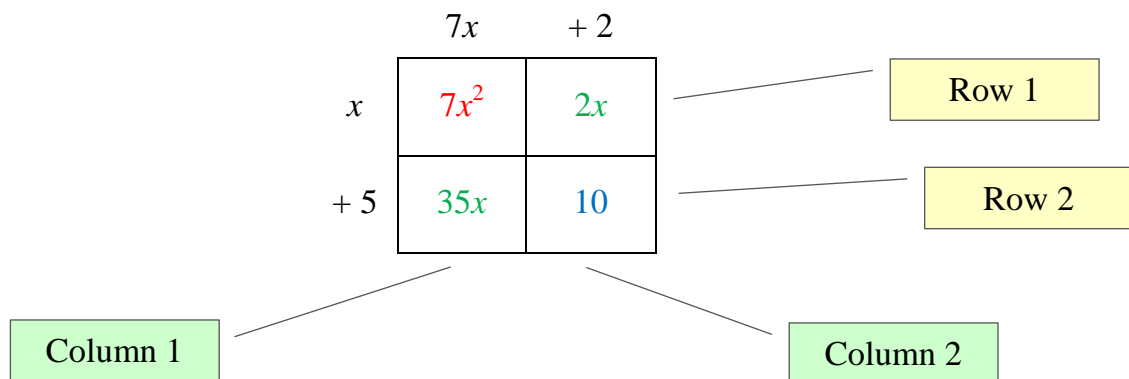
**Step 7:** Find the paired factors of 70 that add up to the middle term 37. They are  $2 \cdot 35$ .

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**Step 8:** Write the factors 2 and 35 in the two remaining boxes. It does not matter which factor goes into which box. Include an 'x' after each factor in the box.

$7x^2$	$2x$
$35x$	10

**Step 9:** Factor out Row 1 then Row 2 to the left side. Factor out Column 1 then Column 2 to the top.



**Step 10:** The resulting factors on the left side and top provide the answer:  $(7x + 2)(x + 5)$ .

**Characteristics of the Box:**

- **Row 1:** Since  $7x^2$  is '+' the sign of the left side factor  $x$  will also be '+'.
- **Row 2:** Since  $35x$  is '+' the sign of the left side factor 5 will also be '+'.
- **Column 1:** Since  $7x^2$  is '+' the sign of the top factor  $7x$  will also be '+'.
- **Column 2:** Since  $2x$  is '+' the sign of the top factor 2 will also be '+'.
- **Miscellaneous:**
  - Since 10 in the bottom right box is '+' the signs of the two terms factored out must be the same: '5' on left side in Row 2 and '2' on top in Column 2. Because the sign of middle term  $37x$  of the trinomial is '+' the signs of both '5' and '2' are also '+', as mentioned in **Step 6**.
  - If the last term of the trinomial is '+' as in this case 10, look at the sign of the middle term  $37x$  to determine the two signs of the binomials in the answer. Since the middle term is '+' the template of the binomial factors in the answer will be  $( + )( + )$ .
  - As a check, the terms factored out on left side and at top can be multiplied together to get the number in the corresponding box. For example, the Column 1 result  $7x$  multiplied by the Row 1 result  $x$  equals  $7x^2$  which is the term in the top-left box.

**CASE 2:** Middle term is '+' and last term is '-'.

$$7x^2 + 33x - 10$$

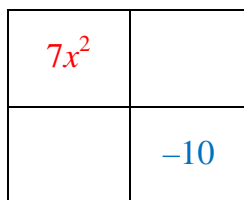
**Step 1:** Factor out any Greatest Common Factors (GCF). None here.

**Step 2:** Ensure a '+' leading coefficient. Factor out '-1' if needed. None here.

**Step 3:** Draw a four-square box.



**Step 4:** Write leading term  $7x^2$  in top-left box. Write last term  $-10$  in bottom-right box.



**Step 5:** Multiply leading coefficient 7 with last term  $-10$  to get  $7 \cdot -10 = -70$ .

**Step 6:** Since the product of  $7 \cdot -10 = -70$ , we know that the signs of the two factors we are seeking must be opposite. List all factors of  $-70$  starting with  $-1 \cdot 70$ , then  $1 \cdot -70$ , and sequentially work up from 1; see table below. There is no need to list factors of 70 in the table that have the same sign, such as  $-1 \cdot -70$ ,  $1 \cdot 70$ , etc. because that will result in a  $+70$  which we do not have in this case.

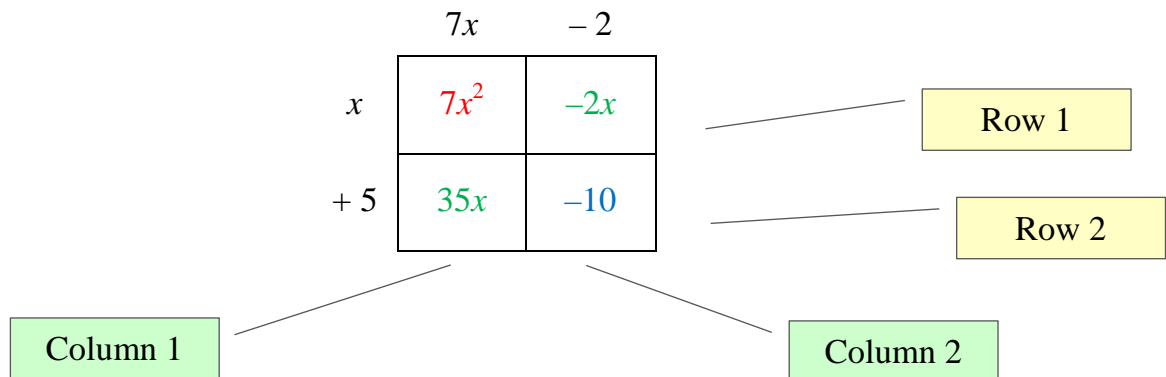
Factors of $-70$	Sum
$-1 \cdot 70$	69
$1 \cdot -70$	-69
$-2 \cdot 35$	33
$2 \cdot -35$	-33
$-5 \cdot 14$	9
$5 \cdot -14$	-9
$-7 \cdot 10$	3
$7 \cdot -10$	-3

**Step 7:** Find the paired factors of  $-70$  that add up to the middle term 33. They are  $-2 \cdot 35$  because the larger of the two factors must be '+' to get 33 when added together.

**Step 8:** Write the factors  $-2$  and  $35$  in the two remaining boxes. It does not matter which factor goes into which box. Include an 'x' after each factor in the box.

$7x^2$	$-2x$
$35x$	$-10$

**Step 9:** Factor out Row 1 then Row 2 to the left side. Factor out Column 1 then Column 2 to the top.



**Step 10:** The resulting factors on the left side and top provide the answer:  $(7x - 2)(x + 5)$ .

**Characteristics of the Box:**

- **Row 1:** Since  $7x^2$  is '+' the sign of the left side factor  $x$  will also be '+'.
- **Row 2:** Since  $35x$  is '+' the sign of the left side factor  $5$  will also be '+'.
- **Column 1:** Since  $7x^2$  is '+' the sign of the top factor  $7x$  will also be '+'.
- **Column 2:** Since  $-2x$  is '-' the sign of the top factor  $2$  will also be '-'.
- **Miscellaneous:**
  - Since  $-10$  in the bottom right box is '-' the signs of the two terms factored out must be opposite: '5' on left side in Row 2 and '-2' on top in Column 2.
  - If the last term of the trinomial is '-' as in this case  $-10$ , the template of the binomial factors in the answer will either be  $( - )( + )$  or  $( + )( - )$ .
  - As a check, the terms factored out on left side and at top can be multiplied together to get the number in the corresponding box. For example, the Column 1 result  $7x$  multiplied by the Row 1 result  $x$  equals  $7x^2$  which is the term in the top-left box.

**CASE 3:** Middle term is  $-37x$  and last term is  $+10$ .

$$7x^2 - 37x + 10$$

**Step 1:** Factor out any Greatest Common Factors (GCF). None here.

**Step 2:** Ensure a '+' leading coefficient. Factor out '-1' if needed. None here.

**Step 3:** Draw a four-square box.


**Step 4:** Write leading term  $7x^2$  in top-left box. Write last term  $10$  in bottom-right box.

$7x^2$	
	$10$

**Step 5:** Multiply leading coefficient 7 with last term 10 to get  $7 \cdot 10 = 70$ .

**Step 6:** List all factors of 70 starting with  $1 \cdot 70$  and sequentially work up from 1; see table below. To determine the signs of the two factors we are seeking in the table, look at the sign of the last term '10' of the trinomial. That sign is '+' which means that both factors have the same sign, either both being '+' or both being '-'. To find which sign, look at the sign of the middle term  $-37x$  of the trinomial. That sign is '-' which means that both factors we are seeking are also '-'. Therefore there is no need to list positive factors of 70 in the table, such as  $1 \cdot 70$ ,  $2 \cdot 35$ , etc.

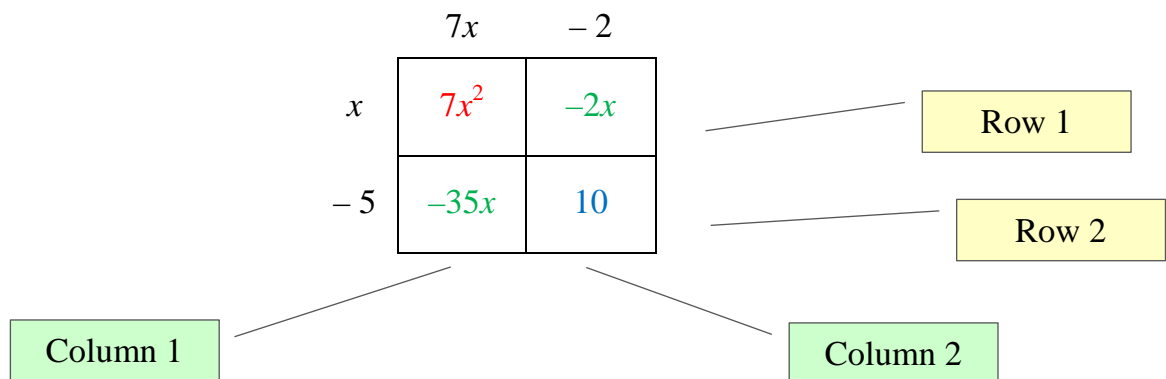
Factors of 70	Sum
$-1 \cdot -70$	$-71$
$-2 \cdot -35$	$-37$
$-5 \cdot -14$	$-19$
$-7 \cdot -10$	$-17$

**Step 7:** Find the paired factors of 70 that add up to the middle term  $-37$ . They are  $-2 \cdot -35$ .

**Step 8:** Write the factors  $-2$  and  $-35$  in the two remaining boxes. It does not matter which factor goes into which box. Include an 'x' after each factor in the box.

$7x^2$	$-2x$
$-35x$	10

**Step 9:** Factor out Row 1 then Row 2 to the left side. Factor out Column 1 then Column 2 to the top.



**Step 10:** The resulting factors on the left side and top provide the answer:  $(7x - 2)(x - 5)$ .

**Characteristics of the Box:**

- **Row 1:** Since  $7x^2$  is '+' the sign of the left side factor  $x$  will also be '+'.
- **Row 2:** Since  $-35x$  is '-' the sign of the left side factor 5 will also be '-'.
- **Column 1:** Since  $7x^2$  is '+' the sign of the top factor  $7x$  will also be '+'.
- **Column 2:** Since  $-2x$  is '-' the sign of the top factor 2 will also be '-'.
- **Miscellaneous:**
  - Since 10 in the bottom right box is '+' the signs of the two terms factored out must be the same: '5' on left side in Row 2 and '2' on top in Column 2. Because the sign of middle term  $-37x$  of the trinomial is '-' the signs of both '5' and '2' are also '-', as mentioned in **Step 6**.
  - If the last term of the trinomial is '+' as in this case +10, look at the sign of the middle term  $-37x$  to determine the two signs of the binomials in the answer. Since the middle term is '-' the template of the binomial factors in the answer will be  $( - )( - )$ .
  - As a check, the terms factored out on left side and at top can be multiplied together to get the number in the corresponding box. For example, the Column 1 result  $7x$  multiplied by the Row 1 result  $x$  equals  $7x^2$  which is the term in the top-left box.

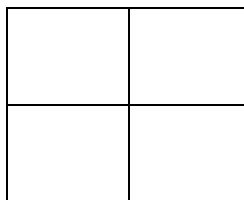
**CASE 4:** Middle term is ' $-33x$ ' and last term is ' $-10$ '.

$$7x^2 - 33x - 10$$

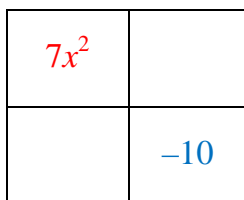
**Step 1:** Factor out any Greatest Common Factors (GCF). None here.

**Step 2:** Ensure a '+' leading coefficient. Factor out '-1' if needed. None here.

**Step 3:** Draw a four-square box.



**Step 4:** Write leading term  $7x^2$  in top-left box. Write last term  $-10$  in bottom-right box.



**Step 5:** Multiply leading coefficient 7 with last term  $-10$  to get  $7 \cdot -10 = -70$ .

**Step 6:** Since the product of  $7 \cdot -10 = -70$ , we know that the signs of the two factors we are seeking must be opposite. List all factors of  $-70$  starting with  $-1 \cdot 70$ , then  $1 \cdot -70$ , and sequentially work up from 1; see table below. There is no need to list factors of 70 in the table that have the same sign, such as  $-1 \cdot -70$ ,  $1 \cdot 70$ , etc. because that will result in a  $+70$  which we do not have in this case.

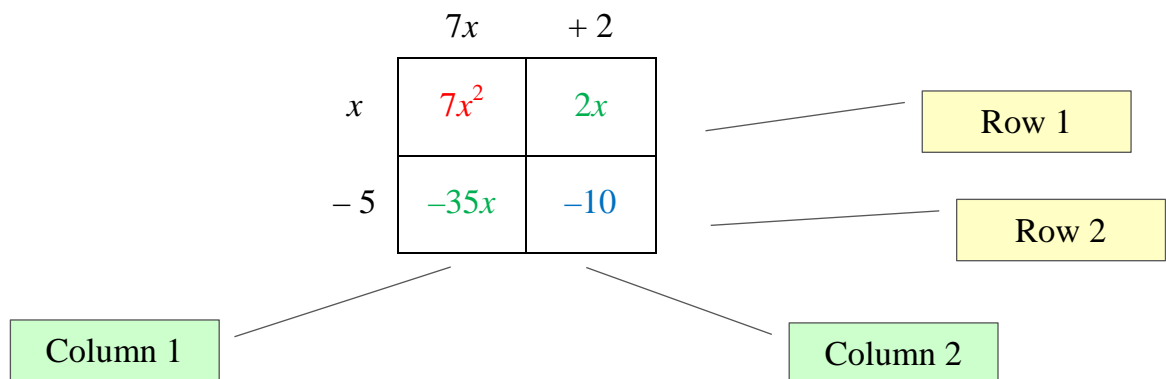
Factors of $-70$	Sum
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$1 \cdot -70$	-69
$-2 \cdot 35$	33
$2 \cdot -35$	-33
$-5 \cdot 14$	9
$5 \cdot -14$	-9
$-7 \cdot 10$	3
$7 \cdot -10$	-3

**Step 7:** Find the paired factors of  $-70$  that add up to the middle term  $-33$ . They are  $2 \cdot -35$  because the larger of the two factors must be '-' to get  $-33$  when added together.

**Step 8:** Write the factors 2 and  $-35$  in the two remaining boxes. It does not matter which factor goes into which box. Include an 'x' after each factor in the box.

$7x^2$	$2x$
$-35x$	$-10$

**Step 9:** Factor out Row 1 then Row 2 to the left side. Factor out Column 1 then Column 2 to the top.



**Step 10:** The resulting factors on the left side and top provide the answer:  $(7x + 2)(x - 5)$ .

**Characteristics of the Box:**

- **Row 1:** Since  $7x^2$  is '+' the sign of the left side factor  $x$  will also be '+'.
- **Row 2:** Since  $-35x$  is '-' the sign of the left side factor 5 will also be '-'.
- **Column 1:** Since  $7x^2$  is '+' the sign of the top factor  $7x$  will also be '+'.
- **Column 2:** Since  $2x$  is '+' the sign of the top factor 2 will also be '+'.
- **Miscellaneous:**
  - Since  $-10$  in the bottom right box is '-' the signs of the two terms factored out must be opposite: '-5' on left side in Row 2 and '2' on top in Column 2.
  - If the last term of the trinomial is '-' as in this case  $-10$ , the template of the binomial factors in the answer will either be  $( - )( + )$  or  $( + )( - )$ .
  - As a check, the terms factored out on left side and at top can be multiplied together to get the number in the corresponding box. For example, the Column 1 result  $7x$  multiplied by the Row 1 result  $x$  equals  $7x^2$  which is the term in the top-left box.

Courtesy of George Hartas

Resources: [www.youtube.com/user/1bshirley](http://www.youtube.com/user/1bshirley) and [www.virtualnerd.com](http://www.virtualnerd.com)