Reverse FOIL Method of Factoring Trinomials

Trinomial Template: $ax^2 + bx + c, \quad a \neq 1$

Example 1: $15x^2 + 11x - 14$

All factor pairs of *a*:

1, 15
3, 5

All factor pairs of *c*:

1, 14
2, 7

3-STEP PROCESS

1. **STEP 1 – Factor Pairs:** List all factor pairs of the first term coefficient *a* (15) and all factor pairs of last the term constant *c* (14). Include positive numbers only, as shown above.
2. **STEP 2 – Mentally Approximate:**
   a. Find that combination of factor pairs of *a* and *c* to multiply, and then to…
   b. Add *or* subtract those products of *a* • *c*, that will…
   c. Result in the middle term *b* (11). Disregard the "+" sign of *b* (+11) for now.

Example of a combination that ***does not result in 11:***

- Try the first factor pairs of *a* (1, 15) and *c* (1, 14).
  - Use **original order** of the numbers within each factor pair of *a* and *c*:
    - Multiply then add: (1 • 1) + (15 • 14) = 211. Not 11.
    - Multiply then subtract: (1 • 1) – (15 • 14) = 209, disregard "–" sign. Not 11.
  - Then **swap order** of the numbers for either *a* or *c*, but ***not both.*** Let's swap *a* to be (15, 1):
    - Multiply then add: (15 • 1) + (1 • 14) = 29. Not 11.
    - Multiply then subtract: (15 • 1) – (1 • 14) = 1. Not 11.
  - There is no combination of the first factor pairs of *a* and *c* that results in 11.

TIPS:

- If *c* is "+" in the trinomial, multiply and ***add only*** in *Step 2*. It would be unnecessary to subtract.
- Keep the *b* value (11) in mind as you scan all factor pairs of *a* with all factor pairs of *c* to see which combination *might* result in 11.
- If it is not apparent which factor pairs of *a* and *c* will result in 11, start with those where the numbers of each factor pair are *closest* to each other. For instance, begin with the following factor pair for *a* (3, 5) and *c* (2, 7). If that combination does not work, then attempt the *next closest* numbers of each factor pair of *a* and/or *c*. Let's try *a* (3, 5) and *c* (2, 7) next.

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Example of a combination that ***results in 11:***

- Try these factor pairs of *a* (3, 5) and *c* (2, 7).
  - Use **original order** of the numbers within each factor pair of *a* and *c*:
    - Multiply then add: (3 • 2) + (5 • 7) = 41. Not 11.
    - Multiply then subtract: (3 • 2) – (5 • 7) = 29, disregard "–" sign. Not 11.
  - Then **swap order** of the numbers for either *a* or *c*, but ***not both.*** Let's swap *a* to be (5, 3):
    - Multiply then add: (5 • 2) + (3 • 7) = 31. Not 11.
    - Multiply then subtract: (5 • 2) – (3 • 7) = 11, disregard "–" sign. Got 11.
    - Go to *Step 3.*

3. **STEP 3 – Binomial Signs**: Now that we found the factor pair combination of *a* and *c* that gives us the 11, we must next determine the signs *inside* of the two binomial factors.
   a. Write the two binomials of the factors of *a* (5, 3) and *c* (2, 7), as shown below.
      i. Version 1: (5 • 2) become *Outers* and (3 • 7) become *Inners* of FOIL multiplication.
      ii. Version 2: (3 • 7) become *Outers* and (5 • 2) become *Inners* of FOIL multiplication.
   b. *Version 2* represents swapping the order of *c* to (7, 2), instead of swapping *a*.
   c. In either version, 11 is obtained by multiplying (5 • 2) and (3 • 7), and then subtracting. Therefore, both versions are equivalent ways to set up the two binomials.

Version 1 or as **Version 2**

(5*x* +or– 7) (3*x* +or– 2) or as (3*x* +or– 2) (5*x* +or– 7)

Version 1	Version 2
+21*x*	–10*x*
–10*x*	+21*x*
+11*x*	+11*x*

The 5 and 3 are from *a*.

The 2 and 7 are from *c*.

   d. Now think what the signs of (5 • 2) and (3 • 7) need to be to get *b* (+11).
      i. The (3 • 7 = 21) ***must*** be positive because 21 is a larger number than (5 • 2 = 10), and we need a +11. This will give us 21 – 10 = +11, as shown in either version above.
      ii. Place the "–" sign in front of the 2 in that binomial. Do not put the "–" sign in front of the 5 because that binomial would then become (–5*x* + 7), which is ***incorrect.*** Do not place a "–" sign in front of the variable term (term with the *x*) of either binomial.
      iii. Finish the problem by writing the correct signs inside of the two binomials.
   e. The trinomial is now factored, with both versions shown below.

Version 1 **Version 2**

(5*x* + 7) (3*x* – 2) or as (3*x* – 2) (5*x* + 7)

NOTE:

- This is a ***moderately difficult*** problem because both *a* and *c* only have a couple of factor pairs each. If instead both *a* and *c* had several factor pairs, as when *a* and *c* are large composite numbers, the process would become much more involved. Let's look at *Example 2* next, a difficult problem.

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Example 2:

$36x^2 - 81x - 40$

All factor pairs of *a*:

1, 36
2, 18
3, 12
4, 9

All factor pairs of *c*:

1, 40
2, 20
4, 10
5, 8

3-STEP PROCESS

1. **STEP 1 – Factor Pairs:** List all factor pairs of the first term coefficient *a* (36) and all factor pairs of last the term constant *c* (40). Include positive numbers only, as shown above.
2. **STEP 2 – Mentally Approximate:**
    a. Find that combination of factor pairs of *a* and *c* to multiply, and then to…
    b. Add *or* subtract those products of *a* • *c*, that will…
    c. Result in the middle term *b* (81). Disregard the "–" sign of *b* (–81) for now.

Example of a combination that ***does not result in 81:***

- Try the first factor pairs of *a* (1, 36) and *c* (1, 40).
    - Use **original order** of the numbers within each factor pair of *a* and *c*:
        - Multiply then add: (1 • 1) + (36 • 40) = 1441. Not 81.
        - Multiply then subtract: (1 • 1) – (36 • 40) = 1439, disregard "–" sign. Not 81.
    - Then **swap order** of the numbers for either *a* or *c*, but ***not both.*** Let's swap *a* to be (36, 1):
        - Multiply then add: (36 • 1) + (1 • 40) = 76. Not 81.
        - Multiply then subtract: (36 • 1) – (1 • 40) = 4, disregard "–" sign. Not 81.
    - There is no combination of the first factor pairs of *a* and *c* that results in 81.

TIPS:

- If *c* is "+" in the trinomial, multiply and ***add only*** in *Step 2*. It would be unnecessary to subtract.
- Keep the *b* value (81) in mind as you scan all factor pairs of *a* with all factor pairs of *c* to see which combination *might* result in 81.
- If it is not apparent which factor pairs of *a* and *c* will result in 81, start with those where the numbers of each factor pair are *closest* to each other. For instance, begin with the following factor pair for *a* (4, 9) and *c* (5, 8). If that combination does not work (it does not in this example), then attempt the *next closest* numbers of each factor pair of *a* and/or *c*. Let's try *a* (3, 12) and *c* (5, 8) next.

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Example of a combination that ***results in 81:***

- Try these factor pairs of *a* (3, 12) and *c* (5, 8).
  - Use **original order** of the numbers within each factor pair of *a* and *c*:
    - Multiply then add: (3 • 5) + (12 • 8) = 111. Not 81.
    - Multiply then subtract: (3 • 5) – (12 • 8) = 81, disregard "–" sign. Got 81.
    - Go to *Step 3.*

3. **STEP 3 – Binomial Signs**: Now that we found the factor pair combination of *a* and *c* that gives us the 81, we must next determine the signs *inside* of the two binomial factors.
   a. Write the two binomials of the factors of *a* (3, 12) and *c* (5, 8), as shown below.
      i. Version 1: (3 • 5) become *Outers* and (12 • 8) become *Inners* of FOIL multiplication.
      ii. Version 2: (12 • 8) become *Outers* and (3 • 5) become *Inners* of FOIL multiplication.
   b. *Version 2* represents swapping the order of both *a* and *c*, to (12, 3) and (8, 5) respectively. However, we should *not swap both* since the same result is obtained with the original order.
   c. In either version, 81 is obtained by multiplying (3 • 5) and (12 • 8), and then subtracting. Therefore, both versions are equivalent ways to set up the two binomials.

Version 1

(3*x* +or– 8) (12*x* +or– 5)

–96*x*
+15*x*
–81*x*

The 3 and 12 are from *a*.

or as

Version 2

(12*x* +or– 5) (3*x* +or– 8)

+15*x*
–96*x*
–81*x*

The 5 and 8 are from *c*.

   d. Now think what the signs of (3 • 5) and (12 • 8) need to be to get *b* (–81).
      i. The (12 • 8 = 96) ***must*** be negative because 96 is a larger number than (3 • 5 = 15), and we need a –81. This will give us 15 – 96 = –81, as shown in either version above.
      ii. Place the "–" sign in front of the 8 in that binomial. Do not put the "–" sign in front of the 12 because that binomial would then become (–12*x* + 5), which is ***incorrect.*** Do not place a "–" sign in front of the variable term (term with the *x*) of either binomial.
      iii. Finish the problem by writing the correct signs inside of the two binomials.
   e. The trinomial is now factored, with both versions shown below.

Version 1: (3*x* – 8) (12*x* + 5) or as **Version 2**: (12*x* + 5) (3*x* – 8)

NOTE:

- This is a ***difficult*** problem because both *a* and *c* have multiple factor pairs, due to *a* (36) and *c* (40) being large composite numbers. If instead *a* and/or *c* are prime numbers, the possible combinations will be considerably less because there will only be one factor pair for *a* and/or *c*, with each factor pair containing just a 1 and that prime number itself.